# Stochastic Discount Factor (2)

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# Outline

- Evidence on limits of arbitrage
- Heterogeneous beliefs
- Short-sales constraints
- Endogenous margin requirements

# Arbitrage in the Real World

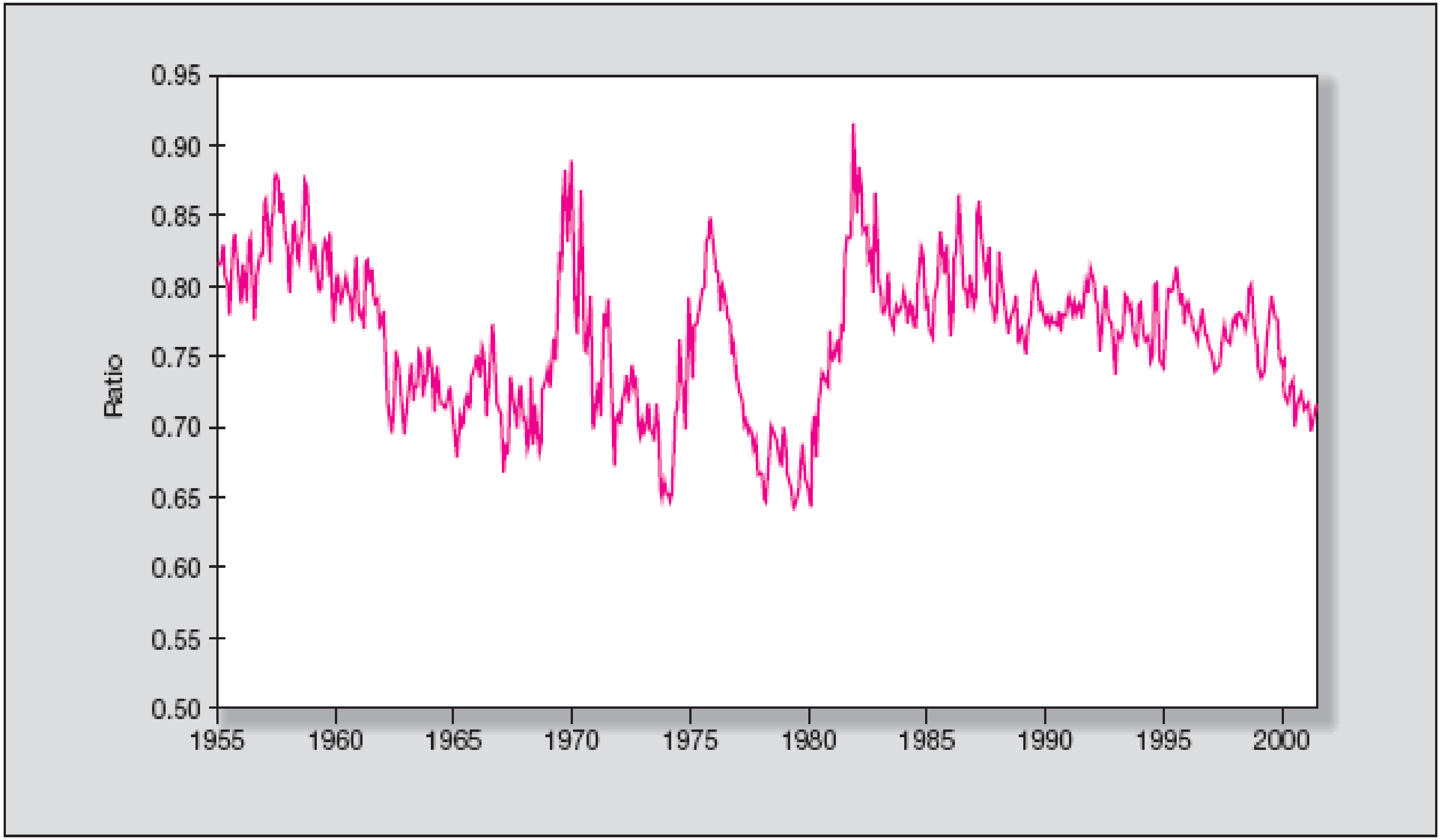
Examples:

- Municipal bonds almost always yield less than Treasury bonds.
- Negative stub values. (One company owns shares in another but trades for less than the market value of those shares, e.g. 3Com/Palm in March 2000.)
- Shares of the same underlying company (Siamese Twins) trading in different locations for different prices (e.g. Royal Dutch/Shell).

What permits this to occur?

# Municipal Bond Yields

**Figure 2.6** **Ratio of yields on tax-exempt to taxable bonds**




Source: Data from Moody's Investors Service.

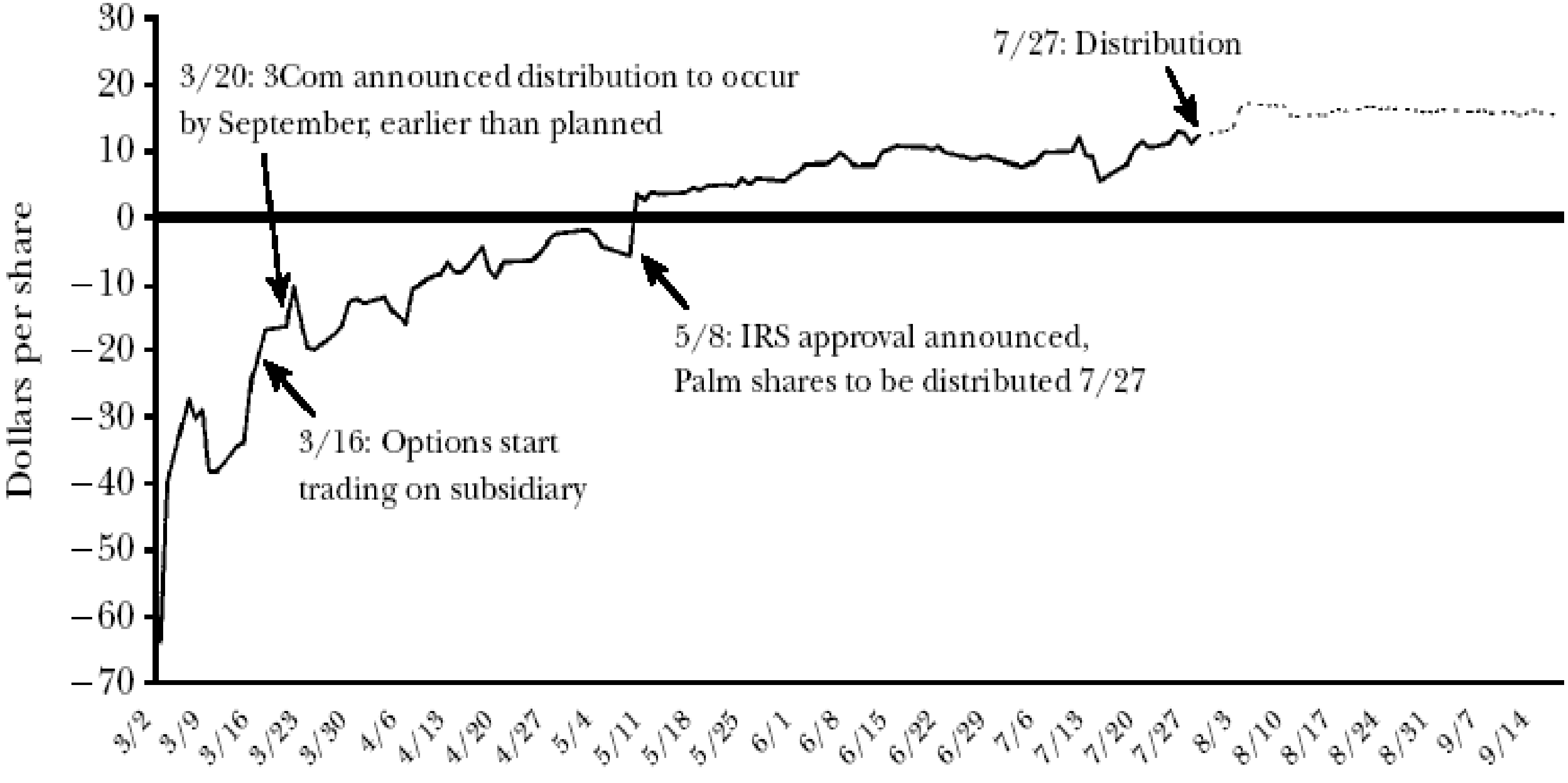
# Municipal Bond Yields

- Municipal bonds pay interest that is exempt from Federal taxes
- It is illegal for investors to short them
- It is illegal for tax-exempt entities to issue tax-free debt in order to hold taxable debt
- This restriction is relevant for Harvard, which can issue tax-free debt to build facilities, but not to add funds to the endowment.

# Negative Stub Values

*Figure 3*

**3Com/Palm Stub, 3/2/00–9/18/00**



# Negative Stub Values

- Negative stub values typically arise in situations where stocks are hard to short, because most investors are individuals who do not know they can profit by lending their shares
- In addition, there are cases where the parent company issues debt using its shareholdings as collateral, then defaults. In these cases holders of the parent company shares lose out. Mitchell, Pulvino, Stafford (Journal of Finance 2002) show that this happens about 30% of the time.
- Arbitrageurs also have to worry about the risk that stub values will become more negative before they turn positive. If this occurs, the arbitrageur will have to put up more collateral. If she can't do this, she will have to close the position at a loss. Example: 12/98 carveout of Ubid by Creative Computers.

# Negative Stub Values

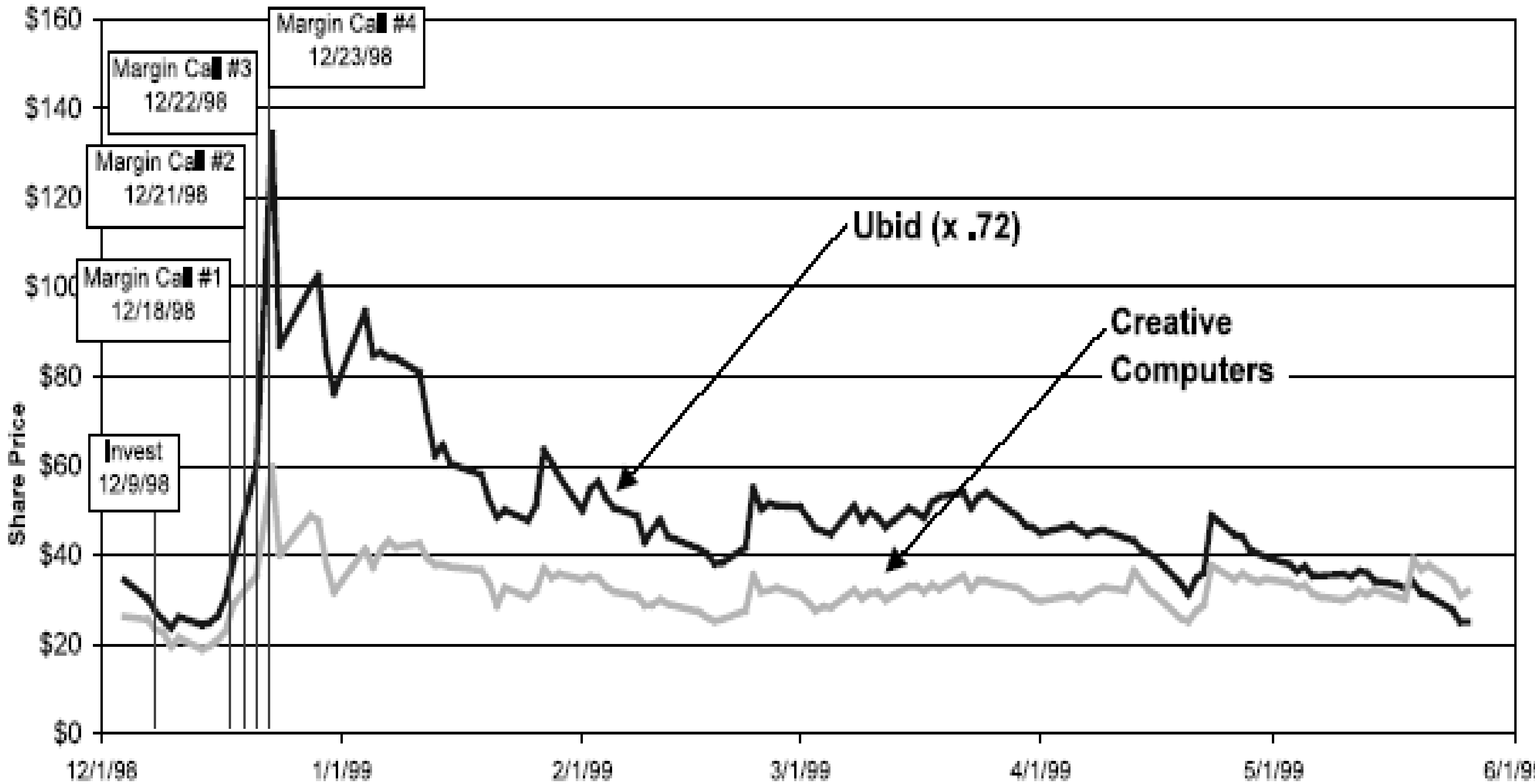


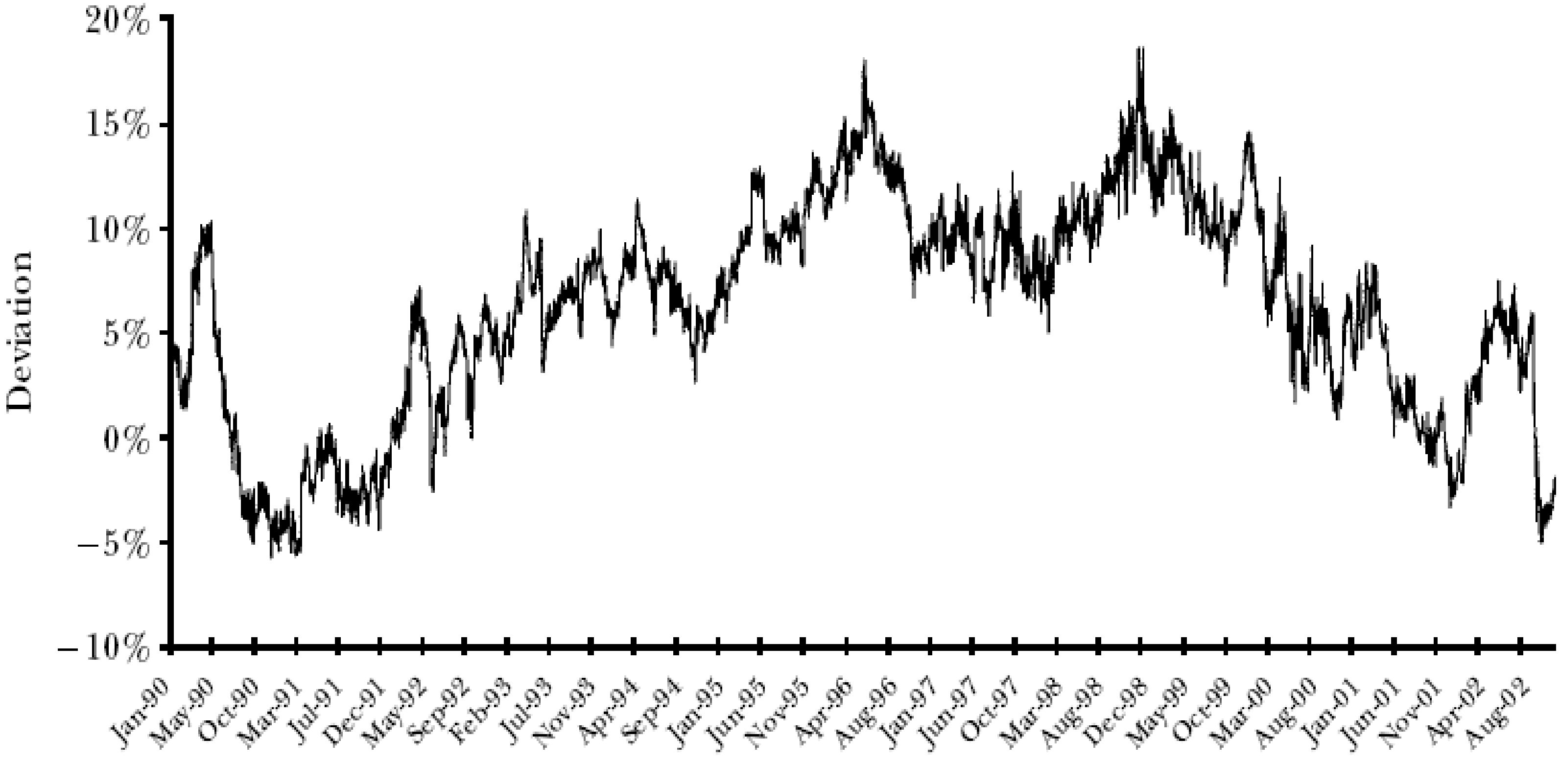

Figure 1. Paths of stock prices for Creative Computers and Ubid.

# Siamese Twins

*Figure 1*

**Pricing of Royal Dutch Relative to Shell**

*(deviation from parity)*



# Siamese Twins

- Prices can be driven apart by preferences of investors for one stock over the other (e.g. because one stock is an index constituent and the other is not).
- The risk that mispricing will worsen before it improves is even more serious in this case, because there is no fixed date at which we can anticipate correct valuation.

# Limits of Arbitrage

- If mispricing worsens it erodes the collateral of arbitrageurs and can force them out of the market
- The exit of arbitrageurs can exacerbate mispricing, causing a "death spiral"
- Shleifer-Vishny "The Limits of Arbitrage" (1997) anticipated the 1998 LTCM crisis
  - 1998 Russian default caused leveraged investors to sell other illiquid assets (EM debt, MBS, etc.)
  - Falling illiquid asset prices caused problems at LTCM
  - LTCM sales and frontrunning drove prices even lower
  - Illiquid asset prices eventually recovered, but too late for LTCM.

# A Discrete-State Model with Heterogeneous Beliefs

Recall the maximization problem of investor $j$,

$$\mathrm{Max}\ u(C_{j0}) + \sum_{s=1}^{S} \beta \pi_j(s) u(C_j(s))$$

subject to

$$C_{j0} + \sum_{s=1}^{S} P_c(s) C_j(s) = Y_{j0} + \sum_{s=1}^{S} P_c(s) Y_j(s).$$

We allow $\pi_j(s)$ to differ across investors, but state prices $P_c(s)$ are given by the market. For simplicity assume all investors have the same utility function.

# A Discrete-State Model with Heterogeneous Beliefs

First-order conditions

$$
\begin{aligned}
u'(C_{j0}) &= \lambda_j \\
\beta\pi_j(s)u'(C_j(s)) &= \lambda_j P_c(s) \text{ for } s = 1...S,
\end{aligned}
$$

where $\lambda_j$ is Lagrange multiplier on budget constraint. Thus

$$
P_c(s) = \frac{\beta\pi_j(s)u'(C_j(s))}{\lambda_j}.
$$

# A Discrete-State Model with Heterogeneous Beliefs

For any two states $s$ and $s^*$ and investors $j$ and $k$,

$$\frac{P_c(s)}{P_c(s^*)} = \frac{\pi_j(s)u'(C_j(s))}{\pi_j(s^*)u'(C_j(s^*))} = \frac{\pi_k(s)u'(C_k(s))}{\pi_k(s^*)u'(C_k(s^*))}.$$

- The investors who have particularly low marginal utility in a state are the ones who give that state the highest probability. These investors perceive wealth in the state as relatively cheap, and buy a lot of it.
- The people who end up rich are those who bet on the state that occurs. "If you're rich, you must be smart" (or lucky).

# Short-Sales Constraints and Heterogeneous Beliefs

Harrison and Kreps (*QJE* 1978) look at the effect of short-sales constraints in a model with heterogeneous beliefs. Their example:

- Dividend $d_t = 0$ or 1. "State $d$" means most recent dividend is $d$.
- Transition probability $q(d, d') = \Pr(d_{t+1} = d' \mid d_t = d)$.
- Transition matrix: Rows denote current state, columns denote future state.

$$Q = \begin{bmatrix} q(0,0) & q(0,1) \\ q(1,0) & q(1,1) \end{bmatrix}.$$

# Present Value Calculations

$$\begin{aligned} E_t d_{t+1} &= 0 \times \Pr(d_{t+1} = 0 \mid d_t) + 1 \times \Pr(d_{t+1} = 1 \mid d_t) \\ &= Q \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{aligned}$$

$$E_t d_{t+i} = Q^i \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$E_t \sum_{i=1}^{\infty} \gamma^i d_{t+i} = [\gamma Q + \gamma^2 Q^2 + \ldots] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \gamma Q [I - \gamma Q]^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

# Heterogeneous Beliefs

- Two classes of investors, 1 and 2. Each has the same $\gamma = 0.75$, but different $Q$.
- $Q^1 = \begin{bmatrix} 1/2 & 1/2 \\ 2/3 & 1/3 \end{bmatrix}$.
- $Q^2 = \begin{bmatrix} 2/3 & 1/3 \\ 1/4 & 3/4 \end{bmatrix}$.
- Who is more optimistic in the initial state 0? Who is more optimistic in the initial state 1?

# Heterogeneous Perceptions of Fundamental Value

- Perceptions of fundamentals can be evaluated by hand (given 2 x 2 matrices).

- $\gamma Q^1[I-\gamma Q^1]^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 12/9 \\ 11/9 \end{bmatrix} = \begin{bmatrix} 1.33 \\ 1.22 \end{bmatrix}.$

- $\gamma Q^2[I-\gamma Q^2]^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 16/11 \\ 21/11 \end{bmatrix} = \begin{bmatrix} 1.45 \\ 1.91 \end{bmatrix}.$

- Does this mean that class 2 investors always hold the stock, and that prices equal class 2 investors' assessment of fundamental value?

# The Option to Resell

- No: with prices 1.45 and 1.91, investor 1 can buy in state 0 and sell in state 1. She perceives the payoff to be

$$[0.75 \times \frac{1}{2} + 0.75^2 \times \left(\frac{1}{2}\right)^2 + ...][1 + 1.91] = 1.75 > 1.45.$$

- But if the price rises to 1.75 in state 0, the price 1.91 is too low in state 1, because investor 2 can buy the asset in state 1 and sell it to investor 1 for 1.75 when state 0 occurs.

- We need a consistent price scheme such that

$$p^*(d_t) = Max_{a=1,2} E^a[\gamma d_{t+1} + \gamma p^*(d_{t+1}) \mid d_t].$$

# Equilibrium Prices

- In this example,

$$\begin{aligned}
p^*(0) &= 0.75 \max \left\{ \frac{1}{2} p^*(0) + \frac{1}{2}[1 + p^*(1)], \frac{2}{3} p^*(0) + \frac{1}{3}[1 + p^*(1)] \right\} \\
&= 1.85.
\end{aligned}$$

$$\begin{aligned}
p^*(1) &= 0.75 \max \left\{ \frac{2}{3} p^*(0) + \frac{1}{3}[1 + p^*(1)], \frac{1}{4} p^*(0) + \frac{3}{4}[1 + p^*(1)] \right\} \\
&= 2.04.
\end{aligned}$$

- Price exceeds fundamental as perceived by either investor. Why?
- Warren Buffett and "Mr. Market".
- Scheinkman-Xiong (*JPE* 2003) derive disagreement from overconfidence and develop testable implications for prices, volume, and free float (available quantity of shares).
- It is possible to get similar effects even without short-sales constraints if agents are risk-averse.

# Short-Sales Constraints in Asset Pricing

- Irrational investors and homogeneous rational investors (behavioral finance literature)
  - "Glamour" stock prices too high
  - Incentive for rational investors to "quasi-short" (create close substitutes to feed the demand)
- Rational investors with heterogeneous hedging needs
  - Diminished risk sharing
  - Precautionary savings
- Rational investors with heterogeneous priors (Harrison-Kreps)
  - "Greater fool" asset demand
  - High trading volume
  - Price can exceed any investor's assessment of fundamental value

# Short-Sales Constraints in Asset Pricing

- Heterogeneous information, rational investors with common priors
  - Some private information doesn't get into prices
  - Investors take this into account in forming their demands
- Heterogeneous information, rational investors with heterogeneous priors (Hong-Stein, *RFS* 2003)
  - Some negative private information doesn't get into prices
  - But other demands don't fully adjust for this
  - When the information is revealed, a crash can occur

# Endogenous Margin Requirement

- The literature on limits of arbitrage initially assumed that the margin requirement for arbitrageurs is constant. When mispricing worsens, arbitrageurs lose wealth and must reduce their positions.
- But what determines the margin requirement?
- If lenders had the same perspective as arbitrageurs, they should be willing to reduce margin requirement to zero (100% financing). Then arbitrageur wealth would have no effect.
- If margin requirements increase, this has a similar effect to a loss of arbitrageur wealth (important in the global financial crisis).
- Geanakoplos (2009) discusses the determination of margins (equivalently, leverage) in equilibrium.

# Endogenous Margin Requirement

- Geanakoplos model has a continuum of beliefs indexed by $h \in (0, 1)$.
- Agent $h$ thinks the probability of a good ("up") state $\pi_U^h = h$, probability of a bad ("down") state $\pi_D^h = 1 - h$.
- Asset $Y$ pays 1 in the $U$ state, 0.2 in the $D$ state.
- All agents hold 1 unit of $Y$, 1 unit of consumption good $C$ at time 0.
- The model rules out short selling, and allows only non-contingent borrowing.

# No Borrowing Equilibrium

- Assume no borrowing. The more optimistic agents buy the asset from the more pessimistic agents. The asset price adjusts to clear the market.
- Equilibrium has $p = 0.68$.
- Marginal buyer $b = 0.60$ values the asset at $0.6 \times 1 + 0.4 \times 0.2 = 0.68$.
- More optimistic agents buy what they can afford: $1/0.68$ units each $\approx 1.5$ units. Total demand is $0.4 \times 1.5 = 0.60$. This is exactly the supply from the pessimistic agents.

# Equilibrium with Riskfree Borrowing

- We allow only non-contingent, non-recourse borrowing. Loan pays $\phi$ in each state. Lender seizes the collateral in the event of default, but has no other recourse.
- Loan payment is $\min(\phi, 1)$ in state $U$ and $\min(\phi, 0.2)$ in state $D$.
- Assume an exogenous constraint $\phi < 0.2\times$ shares held, which makes the debt riskfree.

# Equilibrium with Riskfree Borrowing

- Equilibrium has $p = 0.75$.
- Marginal buyer $b = 0.69$ values the asset at $0.69 \times 1 + 0.31 \times 0.2 = 0.75$.
- More optimistic agents buy what they can afford, and end up holding all of the asset. They spend their initial wealth of 0.31, and borrow 0.20 against their collateral (the unit supply of the risky asset). Their total spending is $0.31 + 0.20 = 0.51$.
- More pessimistic agents sell their asset holdings and lend money to the optimists. Their asset supply is 0.69.
- The price of the asset is amount spent divided by supply, $0.51/0.69 \approx 0.75$.

# Equilibrium with Riskfree Borrowing

- Equilibrium interest rate is zero (no pure time preference, no default risk).
- Margin requirement is $(0.75 - 0.2)/0.75 = 0.73$ per dollar invested.
- Leverage of the optimists is $0.75/(0.75 - 0.2) = 1.36$.
- Equilibrium conditions:

$$
\begin{aligned}
p &= b.1 + (1 - b)0.2 \\
p &= \frac{(1 - b).1 + 0.2}{b}
\end{aligned}
$$

Solve jointly for $p$ and $b$.

# Equilibrium with Risky Borrowing

What if we allow risky borrowing?

Now loan contracts must be indexed by the collateral as well as the amount promised:

$$\text{Contract}_j = (\text{Promise}_j, \text{Collateral}_j) = (A_j, C_j).$$

Contracts are homogeneous of degree one, so we can normalize $C_j = 1$. A riskier loan with lower collateral backing the amount promised is equivalent to a loan with a higher promise given the collateral.

- Write $j$ for promise of $j$ in both states, backed by one share, with price $\pi_j$.
- With no default, $\pi_j = j/(1 + r_f)$. With default, implied interest rate is $1 + r_j = j/\pi_j$.

# Equilibrium with Risky Borrowing

In equilibrium, only riskless borrowing occurs!

Risky loans are priced by the same marginal buyer $b = 0.69$ as before. For example,

$$\begin{aligned} \pi_{0.3} &= 0.69 \times 0.3 + 0.31 \times 0.2 = 0.269. \\ 1 + r_{0.3} &= 0.3/0.269 = 1.12. \end{aligned}$$

Why do optimists not take out risky loans, with lower collateral, to increase their leverage?

# The Absence of Risky Borrowing

- Risky loans shift payments to the good state and away from the bad state. The borrowers think the good state is more likely to occur than the lenders do, so they do not like the price of this shift. In equilibrium, only riskless borrowing occurs.
- The result depends on 2 states at each node of the event tree, and risk-neutral agents with a common discount rate and distinct beliefs.

# How to Get a Crash

- Two-period event tree. Asset pays 1 in states UU, UD, and DU. Two down moves are needed to get payoff of 0.2 in state DD.
- Equilibrium price is 0.95 at the initial node, 1 in state U, and 0.69 in state D.
- The drop in price is greater than any individual agent's drop in perceived fundamental value. Why?

# Mechanisms Amplifying the Effect of Bad News

- Leveraged buyers go bankrupt. Wealth has been redistributed from optimists to pessimists.
- Equilibrium leverage falls from $0.95/(0.95\text{-}0.69) = 3.6$ to $0.69/(0.69\text{-}0.2){=}1.4$. Bad news increases uncertainty, and thus the disagreement between optimists (borrowers) and pessimists (their creditors).

# Disagreement About Bad vs. Good States

- Simsek (2010) considers a model with two types of investors (rational and optimists) and a continuum of states.
- In this model borrowers do sometimes default.
- Key insight is that optimism about bad states (thought less likely to occur) has less effect on asset prices than optimism about good states (thought more likely to occur). Why?

$$P = \left(\frac{1}{1+r}\right)\left\{\pi[v < \overline{v}]\mathrm{E}[v \mid v < \overline{v}] + \pi[v \geq \overline{v}]\mathrm{E}_{OPT}[v \mid v \geq \overline{v}]\right\}.$$

$v$ is future value of asset, $\overline{v}$ is endogenous threshold that triggers default.