

# Present Value and Predictable Returns (2)

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October 2010

# Outline

- Present value relations
  - ▶ Constant discount rates
  - ▶ Linearity generating processes
  - ▶ Gordon growth model
- Rational bubbles
  - ▶ When can they exist?
- Volatility and valuation
  - ▶ Drifting steady state model

# Present Value Relations

- We now turn attention from returns to prices.
- The field of asset pricing ought to have something to say about prices!
- Prices reflect cash flows and discount rates.
- Models of expected returns do not generate predictions for realized return autocorrelations without modelling cash flows, and thus prices.
- The challenge is to find tractable models once we move beyond the simplest assumption of constant discount rates.

# Constant Discount Rates

$$1 + R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}.$$

If the expected return on the stock is constant,  $E_t R_{t+1} = R$ , then

$$P_t = E_t \left[ \frac{P_{t+1} + D_{t+1}}{1 + R} \right].$$

Solving forward for  $K$  periods, we get

$$P_t = E_t \left[ \sum_{k=1}^K \left( \frac{1}{1 + R} \right)^k D_{t+k} \right] + E_t \left[ \left( \frac{1}{1 + R} \right)^K P_{t+K} \right].$$

# Dividend Discount Model

Letting  $K \rightarrow \infty$ , and assuming that the last term on the right hand side converges to zero, we have the *dividend discount model* (DDM) of stock prices,

$$P_t = E_t \left[ \sum_{k=1}^{\infty} \left( \frac{1}{1+R} \right)^k D_{t+k} \right].$$

This model, with a constant expected stock return, is sometimes called the *random walk* or *martingale* model of stock prices. Is this an appropriate name?

# Martingale Model

In fact the stock price is not a martingale in this model, since

$$E_t P_{t+1} = (1 + R)P_t - E_t D_{t+1}.$$

What is a martingale? If we reinvest all dividends in buying more shares, the number of shares we own follows

$$N_{t+1} = N_t \left( 1 + \frac{D_{t+1}}{P_{t+1}} \right).$$

The discounted value of the portfolio,

$$M_t = \frac{N_t P_t}{(1 + R)^t},$$

follows a martingale.

# Linearity-Generating Processes

Gabaix (2009) trick for getting linear relation between prices, dividends, and discount rates. Simplest example, with constant discount rate:

$$E_t D_{t+1} = (1 + g_t) D_t.$$

Assume shocks to  $D_{t+1}$  and shocks to  $g_{t+1}$  are independent of one another. Then

$$E_t D_{t+2} = E_t (1 + g_{t+1}) D_{t+1} = E_t (1 + g_{t+1}) E_t D_{t+1}.$$

Now assume specific functional form for the growth rate:

$$E_t g_{t+1} = \frac{\rho g_t}{1 + g_t}.$$

Almost AR(1), but with "linearity generating twist".

# Linearity-Generating Processes

This functional form implies

$$E_t D_{t+2} = (1 + g_t(1 + \rho))D_t.$$

Expected changes in the dividend decay geometrically at rate  $\rho$ :

$$E_t \Delta D_{t+2} = \rho E_t \Delta D_{t+1}.$$

Thus, with a constant discount rate, we have a special case of the dividend discount model.

Gabaix's contribution is to show that a similar trick can be used more generally, even in models with time-varying discount rates.

# Gordon Growth Model

Named after Myron Gordon, but actually due to John Burr Williams.  
Assume dividends grow at a constant rate  $G$ , so

$$E_t D_{t+k} = (1 + G)^{k-1} E_t D_{t+1}.$$

Then

$$P_t = \frac{E_t D_{t+1}}{R - G},$$

often written as

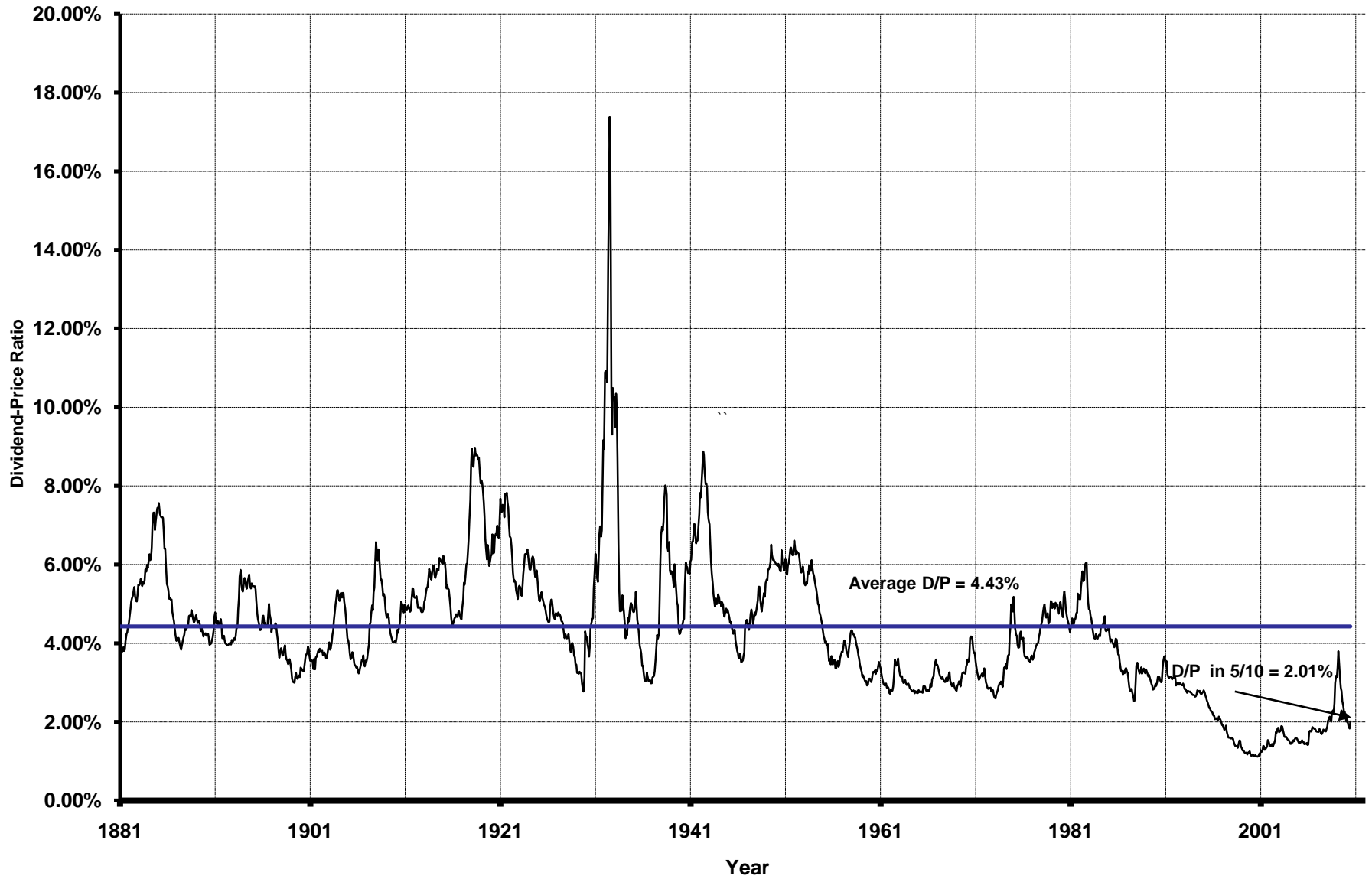
$$\frac{D}{P} = R - G,$$

where  $D$  denotes the next-period dividend. Rearranging this, we have

$$R = \frac{D}{P} + G,$$

which says that returns come from income and capital gains (equal to dividend growth in steady state).

## S&P 500 Dividend/Price



# Steady-State Growth with Earnings

Write earnings as  $X_t$  and the book equity of the firm as  $B_t$ . Then we have

$$B_t = B_{t-1} + X_t - D_t.$$

(This is exactly true under *clean-surplus accounting*, and approximately true under real-world accounting.)

Define *return on equity* (*ROE*) as earnings divided by lagged book equity,  $ROE_t = X_t / B_{t-1}$ .

Define *retention ratio*  $\lambda_t$  as the fraction of earnings that is retained for reinvestment. Then

$$D_t = (1 - \lambda_t)X_t.$$

## Steady-State Growth with Earnings

In the steady state of the Gordon growth model, book equity, earnings, and dividends all grow at the common rate  $G$ . Thus we have

$$G = \frac{B_t - B_{t-1}}{B_{t-1}} = \frac{X_t - D_t}{B_{t-1}} = \lambda \frac{X_t}{B_{t-1}} = \lambda ROE.$$

Substituting into the Gordon formula, we have

$$P_t = \frac{(1 - \lambda)E_t X_{t+1}}{R - \lambda ROE}$$

or

$$\frac{X}{P} = \frac{R - \lambda ROE}{1 - \lambda},$$

where  $X$  denotes next-period earnings.

How does the retention ratio affect the stock price?

## Steady-State Growth with Earnings

$$\frac{X}{P} = \frac{R - \lambda ROE}{1 - \lambda}.$$

Stock price increases with the retention ratio when  $ROE > R$ , and decline with the retention ratio when  $ROE < R$ .

Implied return formula:

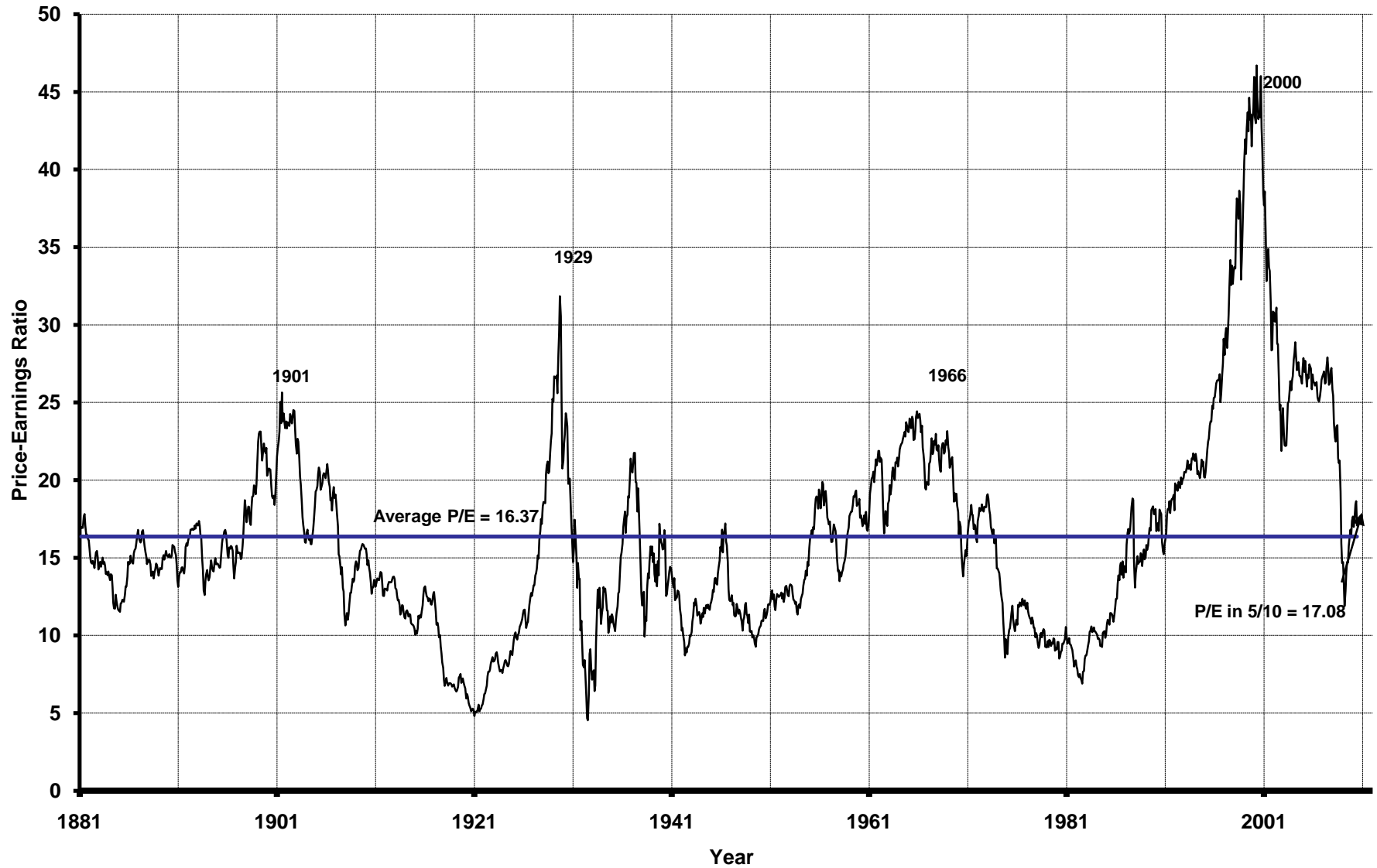
$$R = (1 - \lambda) \frac{X}{P} + \lambda ROE,$$

a weighted average of the earnings yield and profitability.

In the long run, we might expect investments to continue until  $ROE$  is driven to equality with  $R$ . In this case

$$\frac{X}{P} = R = ROE.$$

## S&P 500 Price / 10-Year Average of Earnings



# Rational Bubbles

So far we have assumed that

$$\lim_{K \rightarrow \infty} E_t \left[ \left( \frac{1}{1+R} \right)^K P_{t+K} \right] = 0.$$

Models of rational bubbles violate this assumption. We then get an infinity of possible solutions

$$P_t = P_{Dt} + B_t,$$

where  $P_{Dt}$  is the price in the DDM, and  $B_t$  is any stochastic process satisfying

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# Rational Bubbles

$B_t$  is any stochastic process satisfying

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For example, Blanchard and Watson (1982) suggested the following bubble process:

- $B_{t+1} = ((1 + R)/\pi)B_t + \zeta_{t+1}$  with probability  $\pi$
- $B_{t+1} = \zeta_{t+1}$  with probability  $1 - \pi$
- $E_t \zeta_{t+1} = 0$ .

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- Positive bubbles cannot exist if there is an upper bound on the asset price (e.g. a high-priced substitute in perfectly elastic supply).
- If positive bubbles can exist, but negative bubbles are ruled out, then a bubble can never start. It must exist from the beginning of trading. (Diba and Grossman 1988.)

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- Under the standard assumption that markets are limited only by the OLG structure, this implies that bubbles cannot exist in dynamically efficient economies, because these economies have interest rates higher than growth rates.
- However, recent work by Farhi and Tirole (2008) shows that with incomplete markets and imperfect risksharing, an OLG economy can have a low interest rate (because of precautionary saving) even if it is dynamically efficient. This reconciles dynamic efficiency with the existence of bubbles.

# Useful, Whether They Exist or Not

Even if one does not believe that rational bubbles exist in practice, the rational bubble literature is informative because it tells us what phenomena we may observe in a world of near-rational bubbles (a small amount of persistent return predictability having large effects on prices). Popular discussion of bubbles seems often to be referring to near-rational bubbles.

# Volatility and Valuation

Pastor and Veronesi: Uncertainty about growth rates increases firm value. Consider the Gordon growth model with growth rate uncertain today, but realized over the next period (and constant thereafter):

$$\frac{P}{D} = E \left[ \frac{1}{R - G} \right] > \frac{1}{R - E[G]}$$

by Jensen's Inequality.

# Drifting Steady State Model

When the log dividend-price ratio follows a random walk, we can derive a dynamic version of the Gordon growth model ("drifting steady state model") to see the same effect.

Assume the dividend is known one period in advance:

$$\frac{D_{t+1}}{P_t} = \exp(x_t),$$

Assume that  $x_t$  follows a random walk:

$$x_t = x_{t-1} + \varepsilon_t.$$

# Drifting Steady State Model

Since dividend growth is known one period in advance,

$$\frac{D_{t+1}}{D_t} = 1 + G_t = \exp(g_t).$$

Assume that  $x_{t+1}$  and  $g_{t+1}$  are conditionally normal given time  $t$  information.

The definition of the stock return implies that

$$\begin{aligned} 1 + R_{t+1} &= \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{D_{t+1}}{P_t} + \frac{D_{t+2}}{D_{t+1}} \frac{D_{t+1}}{P_t} \left( \frac{D_{t+2}}{P_{t+1}} \right)^{-1} \\ &= \exp(x_t) [1 + \exp(g_{t+1} - x_{t+1})]. \end{aligned}$$

# Drifting Steady State Model

Use the formula for the conditional expectation of lognormally distributed random variables, and the martingale property that  $E_t x_{t+1} = x_t$ :

$$\begin{aligned} E_t(1 + R_{t+1}) &= \exp(x_t) [1 + E_t \exp(g_{t+1} - x_{t+1})] \\ &= \exp(x_t) [1 + \exp(E_t g_{t+1} - x_t + \sigma_g^2/2 + \sigma_x^2/2 - \sigma_{gx})] \\ &= \frac{D_{t+1}}{P_t} + \exp(E_t g_{t+1}) \exp(\text{Var}_t(p_{t+1} - p_t)/2). \end{aligned}$$

# Drifting Steady State Model

Approximate the RHS using the facts that for small  $y$ ,  $\exp(y) \approx 1 + y$ , and that unexpected log stock returns are approximately equal to unexpected changes in log stock prices:

$$E_t(1 + R_{t+1}) \approx \frac{D_{t+1}}{P_t} + \exp(E_t g_{t+1}) + \frac{1}{2} \text{Var}_t(r_{t+1}).$$

In the original Gordon growth model, returns and dividend growth have the same variance so the model describes both geometric and arithmetic averages.

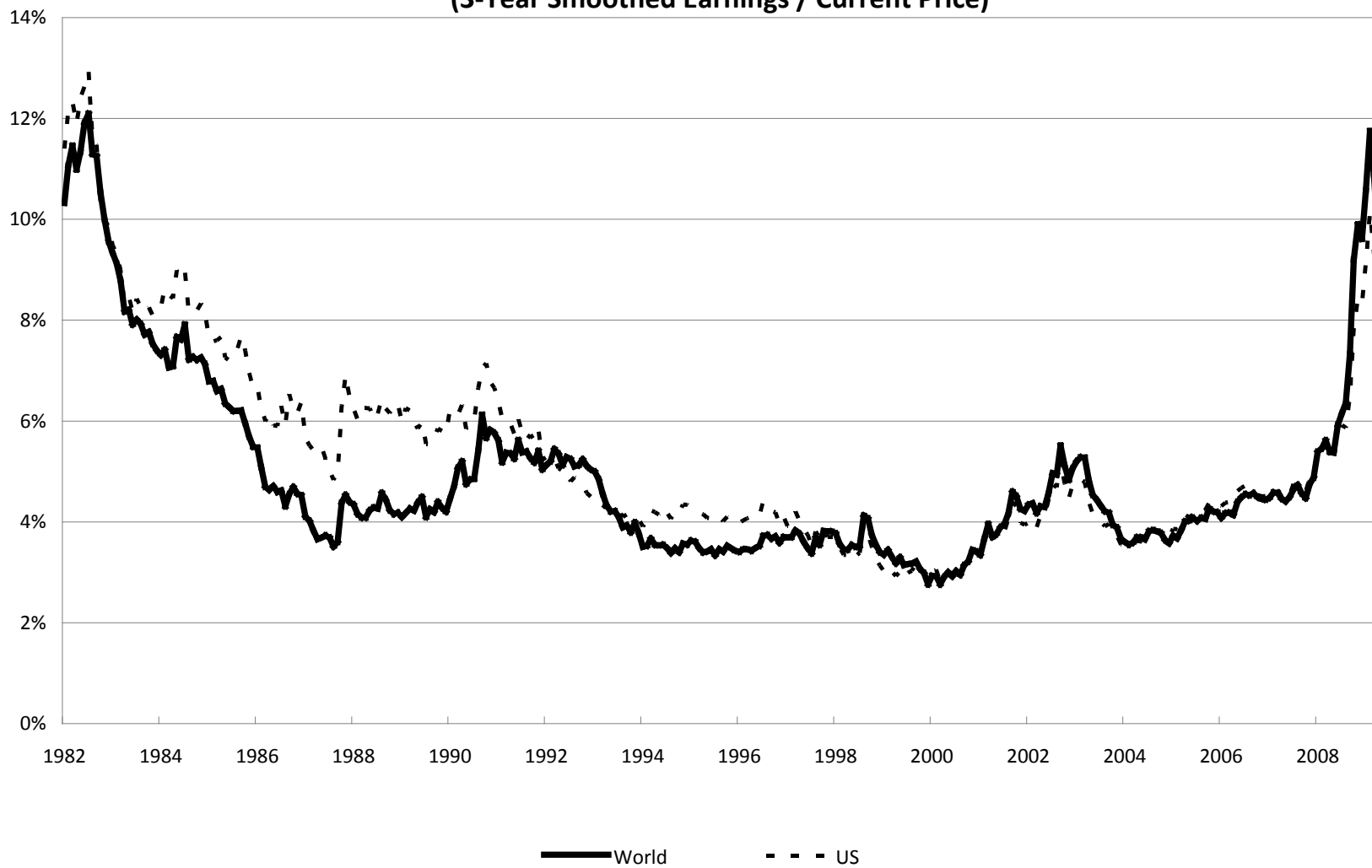
In the dynamic model, it works in the original form for geometric average returns, but we need a variance adjustment for arithmetic average returns.

# Predicting US Stock Returns

- Regressions of returns on information (to be discussed next time) have poor performance out of sample, often doing worse than the historical average return (Goyal and Welch RFS 2008).
- One reason is that the predictive regression has a hard job estimating the intercept.
- Drifting steady state model uses theory to pin down the intercept. Campbell and Thompson (RFS 2008) show that various versions of the approach do better out of sample than the historical average return.
- One implementation, used here: dynamic version of

$$R = (1 - \lambda) \frac{X}{P} + \lambda ROE.$$

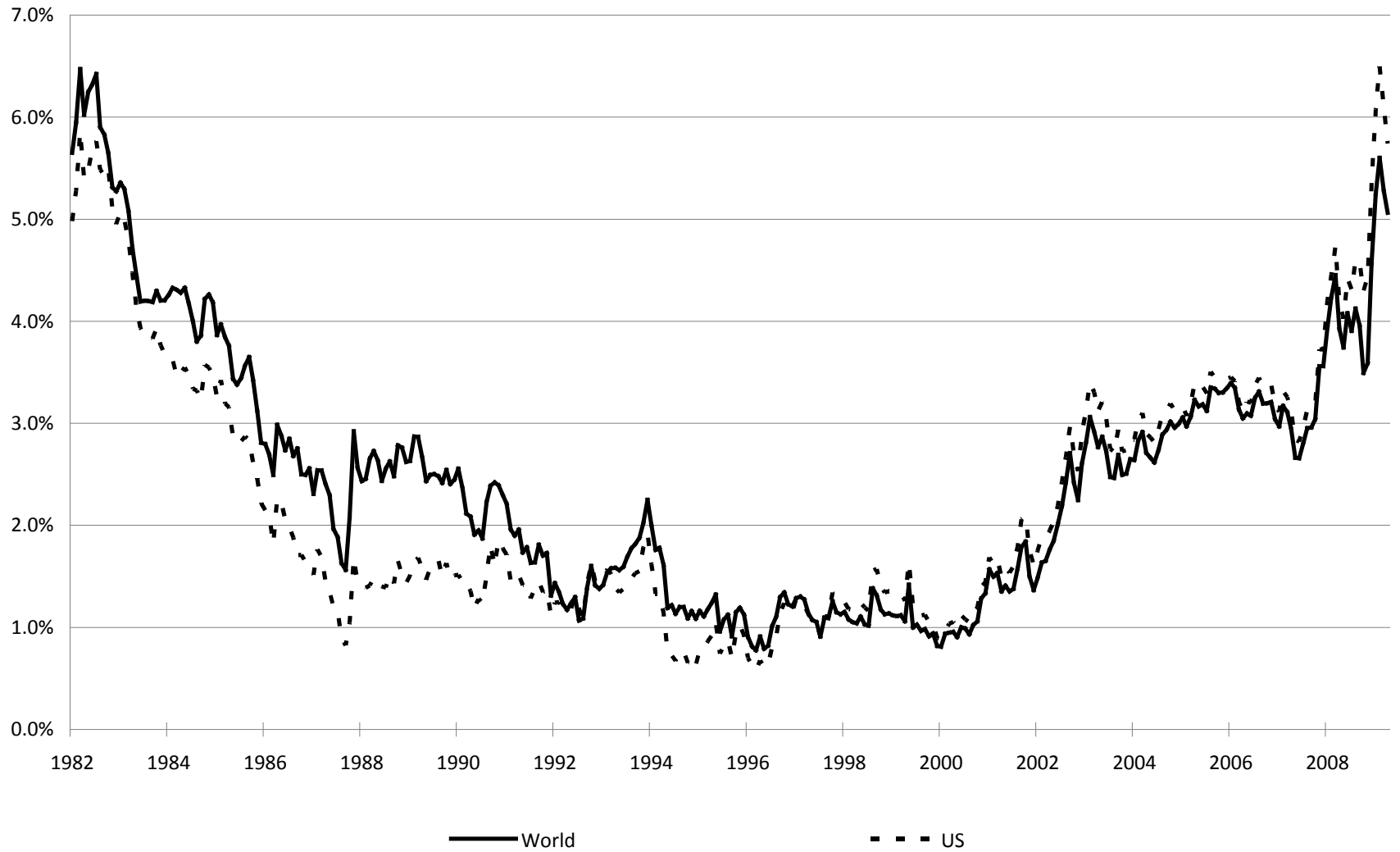
## Smoothed Earnings Yield (3-Year Smoothed Earnings / Current Price)



# Smoothed Real Return on Equity (3-Year Earnings / 3-Year Book Value) - 3-Year Inflation



## Equity Premium



## Inflation-Indexed Government Bond Yields

