Consumption-Based Asset Pricing (1)

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Consumption-Based Asset Pricing

- CBAP is the attempt to relate stock prices to aggregate consumption, which determines marginal utility of a representative agent
- **Equity premium puzzle**:
  - Equity premium is high, which implies a volatile SDF.
  - But consumption is smooth, so we need high curvature of utility function to get volatile SDF.
- **Equity volatility puzzle**: Stock returns are much more volatile than consumption growth.
- **Riskfree rate puzzle**: High risk aversion to explain the equity premium puzzle makes real interest rate very high (or possibly very low!) and highly sensitive to parameters.

Table 1
International stock and bill returns

Country	Sample period	$\overline{r_e}$	$\sigma(r_e)$	$\rho(r_e)$	$\overline{r_f}$	$\sigma(r_f)$	$\rho(r_f)$
USA	1947.2–1998.4	8.085	15.645	0.083	0.896	1.748	0.508
AUL	1970.1–1999.1	3.540	22.699	0.005	2.054	2.528	0.645
CAN	1970.1–1999.2	5.431	17.279	0.072	2.713	1.855	0.667
FR	1973.2–1998.4	9.023	23.425	0.048	2.715	1.837	0.710
GER	1978.4–1997.4	9.838	20.097	0.090	3.219	1.152	0.348
ITA	1971.2–1998.2	3.168	27.039	0.079	2.371	2.847	0.691
JAP	1970.2–1999.1	4.715	21.909	0.021	1.388	2.298	0.480
NTH	1977.2–1998.4	14.070	17.228	–0.030	3.377	1.591	–0.085
SWD	1970.1–1999.3	10.648	23.839	0.022	1.995	2.835	0.260
SWT	1982.2–1999.1	13.744	21.828	–0.128	1.393	1.498	0.243
UK	1970.1–1999.2	8.155	21.190	0.084	1.301	2.957	0.478
USA	1970.1–1998.4	6.929	17.556	0.051	1.494	1.687	0.571
SWD	1920–1998	7.084	18.641	0.096	2.209	5.800	0.710
UK	1919–1998	7.713	22.170	–0.023	1.255	5.319	0.589
USA	1891–1998	7.169	18.599	0.047	2.020	8.811	0.338

Campbell, “Consumption-Based Asset Pricing”, Handbook Chapter 2003

Table 2
International consumption and dividends

Country	Sample period	$\overline{\Delta c}$	$\sigma(\Delta c)$	$\rho(\Delta c)$	$\overline{\Delta d}$	$\sigma(\Delta d)$	$\rho(\Delta d)$
USA	1947.2–1998.4	1.964	1.073	0.216	2.159	28.291	–0.544
AUL	1970.1–1999.1	2.099	2.056	–0.324	0.656	34.584	–0.450
CAN	1970.1–1999.2	2.082	1.971	0.105	–0.488	5.604	0.522
FR	1973.2–1998.4	1.233	2.909	0.029	–0.255	13.108	–0.133
GER	1978.4–1997.4	1.681	2.431	–0.327	1.189	8.932	0.078
ITA	1971.2–1998.2	2.200	1.700	0.283	–3.100	19.092	0.298
JAP	1970.2–1999.1	3.205	2.554	–0.275	–2.350	4.351	0.354
NTH	1977.2–1998.4	1.841	2.619	–0.257	4.679	4.973	0.294
SWD	1970.1–1999.3	0.962	1.856	–0.266	4.977	14.050	0.386
SWT	1982.2–1999.1	0.524	2.112	–0.399	6.052	7.698	0.271
UK	1970.1–1999.2	2.203	2.507	–0.006	0.591	7.047	0.313
USA	1970.1–1998.4	1.812	0.907	0.374	0.612	16.803	–0.578
SWD	1920–1998	1.770	2.816	0.150	1.551	12.894	0.315
UK	1919–1998	1.551	2.886	0.294	1.990	7.824	0.233
USA	1891–1998	1.789	3.218	–0.116	1.516	14.019	–0.087

Representative Agent, Power Utility

Starting point is a representative agent with power utility: time discount factor δ and CRRA γ defined over aggregate consumption C_t.

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}.$$

- When $\gamma = 1$, $U(C_t) = \log(C_t)$.
- Utility is scale-invariant, so risk premia do not alter with aggregate wealth given constant return distributions.
- Investors with different wealth but same CRRA have the same portfolio shares.
- The elasticity of intertemporal substitution or EIS, ψ, is the reciprocal of the CRRA γ. Epstein-Zin utility relaxes this restriction.

SDF with Power Utility

$$U'(C_t) = C_t^{-\gamma}$$

and the SDF is

$$M_{t+1} = \delta(C_{t+1}/C_t)^{-\gamma}.$$

- This is lognormal if consumption is.
- The log SDF is

$$m_{t+1} = \log(\delta) - \gamma \Delta c_{t+1}.$$

Asset Returns Under Lognormality

Assume joint lognormality and homoskedasticity of asset returns and consumption. Expected returns are given by

$$0 = \mathrm{E}_t r_{i,t+1} + \log\delta - \gamma \mathrm{E}_t \Delta c_{t+1} + \left(\frac{1}{2}\right)\left[\sigma_i^2 + \gamma^2\sigma_c^2 - 2\gamma\sigma_{ic}\right] .$$

Here σ_c^2 denotes the unconditional variance of log consumption innovations $\mathrm{Var}(c_{t+1} - \mathrm{E}_t c_{t+1})$, and σ_{ic} denotes the unconditional covariance of innovations $\mathrm{Cov}(r_{i,t+1} - \mathrm{E}_t r_{i,t+1},\ c_{t+1} - \mathrm{E}_t c_{t+1})$.
The riskfree rate is

$$r_{f,t+1} = -\log\delta + \gamma \mathrm{E}_t \Delta c_{t+1} - \frac{\gamma^2\sigma_c^2}{2} .$$

The risk premium on any other asset is

$$\mathrm{E}_t\left[r_{i,t+1} - r_{f,t+1}\right] + \frac{\sigma_i^2}{2} = \gamma\sigma_{ic} .$$

Equity Premium Puzzle

Empirically, σ_{ic} is low for stocks. Thus γ must be large to fit the high average returns on stocks.
We can write the consumption covariance as

$$\sigma_{ic} = \sigma_i \sigma_c \rho_{ic},$$

where ρ_{ic} is the consumption correlation. Empirically, ρ_{ic} is low but even if we set it to one we still do not bring γ down to a reasonable level.

Table 4
The equity premium puzzle

Country	Sample period	$\overline{aer_e}$	$\sigma(er_e)$	$\sigma(m)$	$\sigma(\Delta c)$	$\rho(er_e,\Delta c)$	$\mathrm{cov}(er_e,\Delta c)$	RRA(1)	RRA(2)
USA	1947.2–1998.3	8.071	15.271	52.853	1.071	0.205	3.354	240.647	49.326
AUL	1970.1–1998.4	3.885	22.403	17.342	2.059	0.144	6.640	58.511	8.421
CAN	1970.1–1999.1	3.968	17.266	22.979	1.920	0.202	6.694	59.266	11.966
FR	1973.2–1998.3	8.308	23.175	35.848	2.922	–0.093	–6.315	< 0	12.270
GER	1978.4–1997.3	8.669	20.196	42.922	2.447	0.029	1.446	599.468	17.542
ITA	1971.2–1998.1	4.687	27.068	17.314	1.665	–0.006	–0.252	< 0	10.400
JAP	1970.2–1998.4	5.098	21.498	23.715	2.561	0.112	6.171	82.620	9.260
NTH	1977.2–1998.3	11.421	16.901	67.576	2.510	0.032	1.344	849.991	26.918
SWD	1970.1–1999.2	11.539	23.518	49.066	1.851	0.015	0.674	1713.197	26.501
SWT	1982.2–1998.4	14.898	21.878	68.098	2.123	–0.112	–5.181	< 0	32.076
UK	1970.1–1999.1	9.169	21.198	43.253	2.511	0.093	4.930	185.977	17.222
USA	1970.1–1998.3	6.353	16.976	37.425	0.909	0.274	4.233	150.100	41.178
SWD	1920–1997	6.540	18.763	34.855	5.622	0.167	8.830	74.062	12.400
UK	1919–1997	8.674	21.277	40.767	5.630	0.351	21.042	41.223	14.483
USA	1891–1997	6.723	18.496	36.345	6.437	0.495	29.450	22.827	11.293

Reactions to the Equity Premium Puzzle (1)

- Risk aversion is high. But this creates a riskfree rate puzzle because the average riskfree rate is

$$\mathrm{E}r_{f,t+1} = -\log\delta + \gamma \mathrm{E}\Delta c_{t+1} - \frac{\gamma^2\sigma_c^2}{2}$$

  which is poorly behaved when γ is large.
- A confidence interval for γ includes reasonable values.
- Average returns on stocks are overstated because
  - We ignore taxation (McGrattan and Prescott).
  - US returns were unusually high, or 20th Century returns were unusually high (peso problem, see Dimson, Marsh, and Staunton).

Table 5
The risk-free rate puzzle

Country	Sample period	$\overline{r_f}$	$\overline{\Delta c}$	$\sigma(\Delta c)$	RRA(1)	TPR(1)	RRA(2)	TPR(2)
USA	1947.2–1998.3	0.896	1.951	1.071	240.647	–136.270	49.326	–81.393
AUL	1970.1–1998.4	2.054	2.071	2.059	58.511	–46.512	8.421	–13.880
CAN	1970.1–1999.1	2.713	2.170	1.920	59.266	–61.154	11.966	–20.618
FR	1973.2–1998.3	2.715	1.212	2.922	< 0	N/A	12.270	–5.735
GER	1978.4–1997.3	3.219	1.673	2.447	599.468	9757.265	17.542	–16.910
ITA	1971.2–1998.1	2.371	2.273	1.665	< 0	N/A	10.400	–19.765
JAP	1970.2–1998.4	1.388	3.233	2.561	82.620	–41.841	9.260	–25.735
NTH	1977.2–1998.3	3.377	1.671	2.510	849.991	21349.249	26.918	–18.769
SWD	1970.1–1999.2	1.995	1.001	1.851	1713.197	48590.956	26.501	–12.506
SWT	1982.2–1998.4	1.393	0.559	2.123	< 0	N/A	32.076	6.636
UK	1970.1–1999.1	1.301	2.235	2.511	185.977	676.439	17.222	–27.838
USA	1970.1–1998.3	1.494	1.802	0.909	150.100	–175.916	41.178	–65.701
SWD	1920–1997	2.209	1.730	2.811	74.062	90.793	12.400	–13.165
UK	1919–1997	1.255	1.472	2.815	41.223	7.913	14.483	–11.749
USA	1891–1997	2.020	1.760	3.218	22.827	–11.162	11.293	–11.247

Reactions to the Equity Premium Puzzle (2)

- Consumption growth is not lognormal, and true expected returns are high because of a small probability of a disaster (Barro) or parameter uncertainty (Weitzman).
- The short-run covariance with consumption does not adequately represent long-run consumption risk because
  - There are adjustment costs in consumption (Gabaix-Laibson), or
  - Consumption growth has a persistent component and consumers have Epstein-Zin utility (Bansal-Yaron).

Reactions to the Equity Premium Puzzle (3)

- The representative agent model is flawed because
  - consumers have idiosyncratic, uninsurable labor income risk
  - not all consumers participate in the stock market
  - some consumers are borrowing constrained.
- The power utility model does not adequately represent preferences. Alternatives:
  - Epstein-Zin utility
  - Habit formation utility (Constantinides, Campbell-Cochrane).