

# Consumption-Based Asset Pricing (2)

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# Outline

- What if consumption is not lognormal?
  - ▶ Rietz (1988) "disaster risk" explanation for equity premium revived by Barro (2006)
  - ▶ Equity premium is high because higher moments contribute to risk
  - ▶ Martin (2010) treatment of asset pricing with iid consumption growth but arbitrary higher moments
- What if we relax the assumption of power utility that risk aversion is the reciprocal of the elasticity of intertemporal substitution?
  - ▶ Epstein-Zin (1989) preferences
  - ▶ Substituting out consumption or wealth to get CAPM+ and CCAPM+ models
  - ▶ Effects of persistent consumption growth and changing variance within a lognormal model
  - ▶ Concluding thoughts on time-varying disaster risk

# Non-lognormal Consumption

- Assume power utility with time discount factor  $\delta$  and risk aversion  $\gamma$ .
- Consider an asset that pays  $D_t = C_t^\lambda$ .
- The parameter  $\lambda$  scales the volatility of dividends (a proxy for leverage).
  - ▶ When  $\lambda = 0$ , the asset is riskless.
  - ▶ When  $\lambda = 1$ , the asset is the aggregate wealth portfolio which pays aggregate consumption.

$$\begin{aligned} P_t &= E_t \sum_{j=1}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{-\gamma} C_{t+j}^\lambda \\ &= D_t E_t \sum_{j=1}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{\lambda-\gamma}. \end{aligned}$$

# Non-lognormal Consumption

- Define  $\delta = \exp(-r^*)$ , so  $r^*$  is the pure rate of time preference.
- Assume iid consumption growth and define  $G = c_{t+1} - c_t$ .

$$\begin{aligned} P_t &= D_t E_t \sum_{j=1}^{\infty} \delta^j \left( \frac{C_{t+j}}{C_t} \right)^{\lambda - \gamma} \\ &= D_t \sum_{j=1}^{\infty} \exp(-r^* j) E[(\exp(\lambda - \gamma) G)^j]. \end{aligned}$$

## Cumulant Generating Function

The cumulant generating function for any random variable  $G$  is the log of the moment generating function:

$$c(\theta) = \log E \exp(\theta G).$$

(Note  $c$  does not refer to log consumption here!)

Important property:

$$c(\theta) = \sum_{n=1}^{\infty} \frac{\kappa_n \theta^n}{n!},$$

where  $\kappa_n$  is the  $n$ 'th cumulant of  $G$ .

- Here,  $\kappa_1$  is the mean of log consumption growth,  $\kappa_2$  is the variance  $\sigma^2$ ,  $\kappa_3/\sigma^3$  is the skewness,  $\kappa_4/\sigma^4$  is the excess kurtosis, and so forth.
- All cumulants above the second are zero when log consumption growth is normal.
- $c(0) = 0$  and  $c(1)$  is the log of the mean of simple gross consumption growth.

# Dividend-Price Ratio

$$\begin{aligned}
 P_t &= D_t \sum_{j=1}^{\infty} \exp(-r^* j) E[(\exp(\lambda - \gamma) G)^j] \\
 &= D_t \sum_{j=1}^{\infty} \exp[-(r^* - c(\lambda - \gamma))j] \\
 &= D_t \frac{\exp[-(r^* - c(\lambda - \gamma))]}{1 - \exp[-(r^* - c(\lambda - \gamma))]} .
 \end{aligned}$$

Define  $d/p = \log(1 + D_t/P_t)$ , the log gross dividend yield. Then

$$d/p = r^* - c(\lambda - \gamma).$$

Special case: when  $\lambda = 1$ , we have a consumption claim and

$$c/w = r^* - c(1 - \gamma) = r^* - \sum_{n=1}^{\infty} \frac{\kappa_n (1 - \gamma)^n}{n!}.$$

# Gross Return

The gross return on the asset is

$$\begin{aligned}1 + R_{t+1} &= \frac{P_{t+1}}{P_t} \left( 1 + \frac{D_{t+1}}{P_{t+1}} \right) \\ &= \frac{D_{t+1}}{D_t} \exp(r^* - c(\lambda - \gamma)).\end{aligned}$$

Thus the expected gross return is

$$\begin{aligned}1 + \mathbb{E}R_{t+1} &= \mathbb{E} \exp(G\lambda) \exp(r^* - c(\lambda - \gamma)) \\ &= \exp(r^* - c(\lambda - \gamma) + c(\lambda)).\end{aligned}$$

Define  $er = \log(1 + \mathbb{E}R_{t+1})$ , the log of the expected gross return. Then

$$er = r^* - c(\lambda - \gamma) + c(\lambda).$$

# Equity Premium

$$er = r^* - c(\lambda - \gamma) + c(\lambda).$$

Special cases:

- When  $\lambda = 0$ , we have a riskless asset and

$$r_f = r^* - c(-\gamma) = r^* - \sum_{n=1}^{\infty} \frac{\kappa_n (-\gamma)^n}{n!}.$$

- When  $\lambda = 1$ , we have a consumption claim and

$$er = r^* - c(1 - \gamma) + c(1).$$

- The risk premium on the consumption claim (the equity premium) is the difference:

$$rp = c(1) + c(-\gamma) - c(1 - \gamma) = \sum_{n=2}^{\infty} \frac{\kappa_n}{n!} \{1 + (-\gamma)^n - (1 - \gamma)^n\}.$$

These results generalize the familiar lognormal formulas to allow for the influence of higher moments.

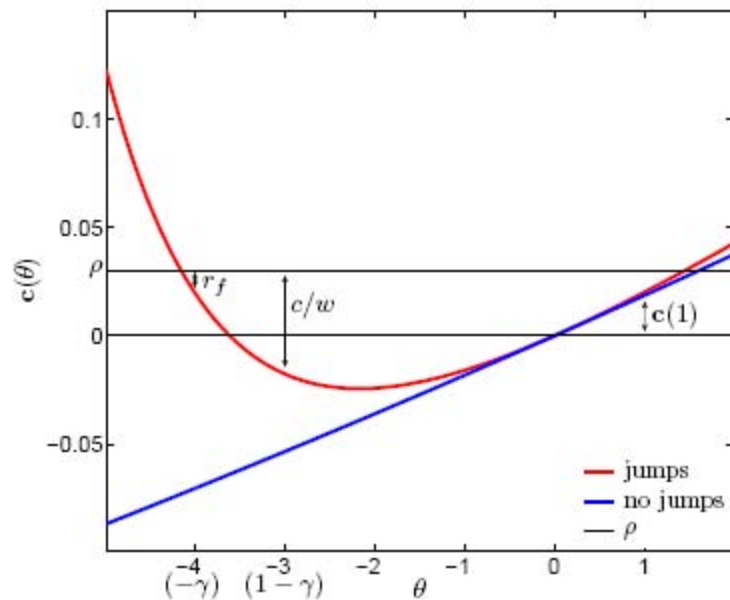
# Gordon Growth Model

Putting these results together, we have a Gordon growth model,

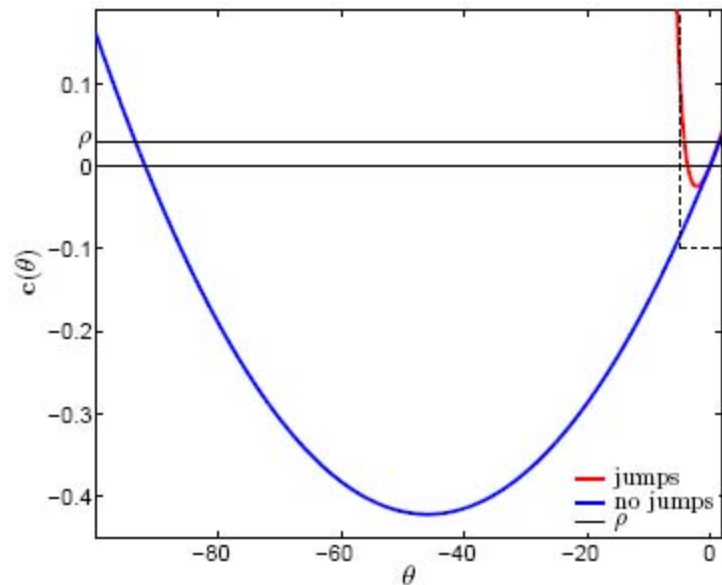
$$dp = er - c(\lambda).$$

In the case of the consumption claim,

$$c/w = r_f + rp - c(1).$$



(a)  $r_f$ ,  $c/w$ , and  $c(1)$  can be read off the CGF



(b) Zooming out

Figure 1: Left: The CGF in equation (18) shown with and without ( $\omega = 0$ ) jumps. The figure assumes that  $\gamma = 4$ . Right: Zooming out to see the equity premium and riskless rate puzzles. The dashed box in the upper right-hand corner indicates the region plotted in Figure 1a.

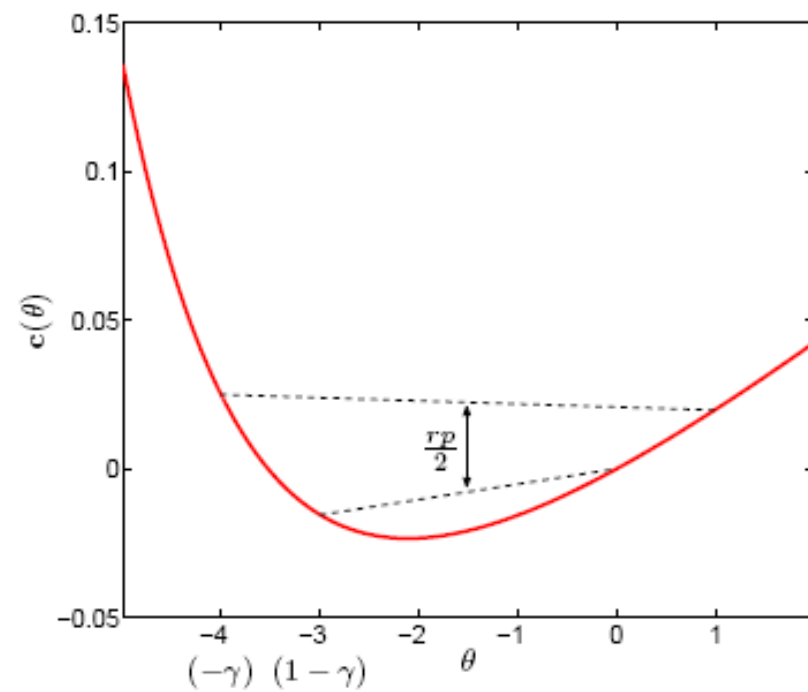


Figure 2: The risk premium. The figure assumes that  $\gamma = 4$ .

	$\omega$	$b$	$s$	$R_f$	$C/W$	$RP$	$R_f^*$	$C/W^*$	$RP^*$
<b>Baseline case</b>	<b>0.017</b>	<b>0.39</b>	<b>0.25</b>	<b>1.0</b>	<b>4.8</b>	<b>5.7</b>	<b>-0.9</b>	<b>2.8</b>	<b>5.7</b>
High $\omega$	0.022			-2.4	3.1	7.4	-2.5	3.0	7.4
Low $\omega$	0.012			4.5	6.4	4.1	0.7	2.6	4.1
High $b$		0.44		-1.9	3.6	7.5	-2.6	2.9	7.5
Low $b$		0.34		3.5	5.8	4.4	0.4	2.7	4.4
High $s$			0.30	-2.2	3.8	8.1	-3.1	2.9	8.1
Low $s$			0.20	3.2	5.5	4.2	0.5	2.7	4.2

Table I: The impact of different assumptions about the distribution of disasters.  $\tilde{\mu} = 0.025$ ,  $\sigma = 0.02$ . Unasterisked group assumes power utility,  $\rho = 0.03$ ,  $\gamma = 4$ . Asterisked group assumes Epstein-Zin preferences,  $\rho = 0.03$ ,  $\gamma = 4$ ,  $\psi = 1.5$ .

$n$	$R_f$	$C/W$	$RP$	
1	10.3	8.5	0.0	deterministic
2	7.1	6.7	1.6	lognormal
3	4.7	5.7	3.0	
4	3.0	5.1	4.1	
$\infty$	1.0	4.8	5.7	true model

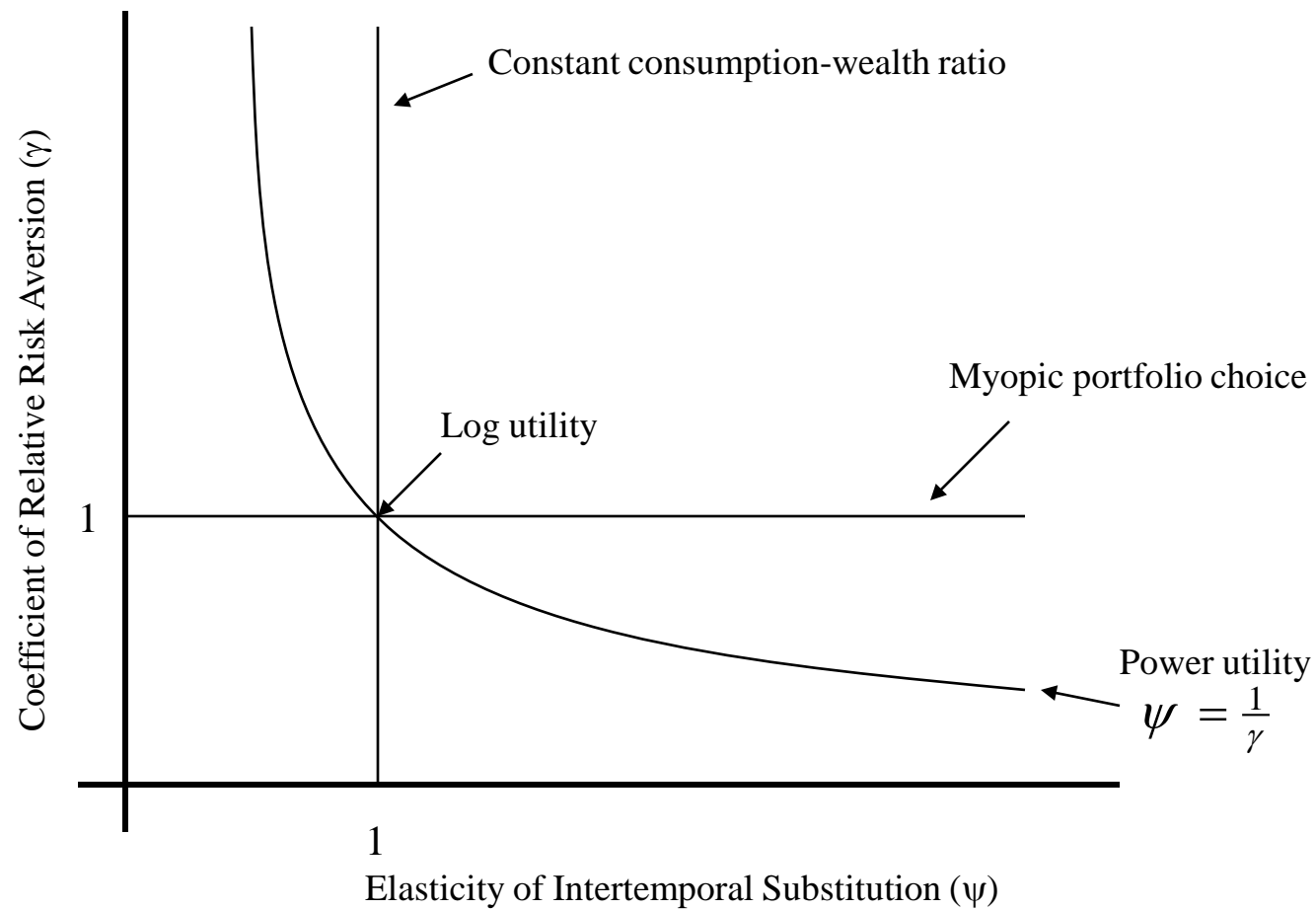
Table II: The impact of approximating the disaster model by truncating at the  $n$ th cumulant. All parameters as in baseline power utility case of Table I.

# Epstein-Zin Preferences

$$U_t = \left\{ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}},$$

where  $\theta \equiv (1 - \gamma)/(1 - 1/\psi)$ .

- Here  $\gamma$  is risk aversion and  $\psi$  is the elasticity of intertemporal substitution.
- When  $\gamma = 1/\psi$ ,  $\theta = 1$  and the recursion becomes linear; it can then be solved forward to yield the familiar time-separable power utility model.



# Euler Equation

Assume intertemporal budget constraint

$$W_{t+1} = (1 + R_{w,t+1}) (W_t - C_t).$$

Then we get an Euler equation

$$1 = E_t \left[ \left\{ \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right\}^{\theta} \left\{ \frac{1}{(1 + R_{w,t+1})} \right\}^{1-\theta} (1 + R_{i,t+1}) \right].$$

- Different from power utility because the Euler equation depends on the form of the intertemporal budget constraint.
- All assets must be tradable and included in wealth.

# Lognormal Version of Epstein-Zin Model

If asset returns and consumption are homoskedastic and jointly lognormal,

$$r_{f,t+1} = -\log \delta + \frac{1}{\psi} E_t[\Delta c_{t+1}] + \frac{\theta - 1}{2} \sigma_w^2 - \frac{\theta}{2\psi^2} \sigma_c^2.$$

$$E_t[r_{i,t+1}] - r_{f,t+1} + \frac{\sigma_i^2}{2} = \theta \frac{\sigma_{ic}}{\psi} + (1 - \theta) \sigma_{iw}.$$

- The Epstein-Zin model nests the consumption CAPM with power utility ( $\theta = 1$ ) and the traditional static CAPM ( $\theta = 0$ ).
- But can we treat  $\sigma_{ic}$  and  $\sigma_{iw}$  as independently measurable quantities?

# Approximate Budget Constraint

$$r_{w,t+1} - E_t r_{w,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{w,t+1+j} \\ - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}.$$

- $d_{w,t} = c_t$
- $E_t r_{w,t+1} = (1/\psi) E_t [\Delta c_{t+1}]$

$$r_{w,t+1} - E_t r_{w,t+1} = (\Delta c_{t+1} - E_t \Delta c_{t+1}) \\ + \left(1 - \frac{1}{\psi}\right) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}.$$

# Substituting Out Consumption

$$\Delta c_{t+1} - E_t \Delta c_{t+1} = r_{w,t+1} - E_t r_{w,t+1} \\ + (1 - \psi)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}.$$

$$\sigma_{ic} = \sigma_{iw} + (1 - \psi)\sigma_{ih},$$

$$\sigma_{ih} \equiv \text{Cov}(r_{i,t+1} - E_t r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}).$$

# Substituting Out Consumption

$$E_t[r_{i,t+1}] - r_{f,t+1} + \frac{\sigma_i^2}{2} = \gamma\sigma_{iw} + (\gamma - 1)\sigma_{ih}.$$

- Call this “CAPM+”, because it nests the CAPM and adds aversion to changing investment opportunities.
- We get CAPM when  $\gamma = 1$  (myopic asset demand).
- The EIS  $\psi$  plays no direct role.
- Empirical implementation of Merton (1973) intertemporal CAPM (ICAPM) due to Campbell (1993).

# Substituting Out Wealth

$$\sigma_{iw} = \sigma_{ic} + \left(1 - \frac{1}{\psi}\right) \sigma_{ig},$$

$$\sigma_{ig} \equiv \text{Cov}(r_{i,t+1} - E_t r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}).$$

$$E_t[r_{i,t+1}] - r_{f,t+1} + \frac{\sigma_i^2}{2} = \gamma \sigma_{ic} + \left(\gamma - \frac{1}{\psi}\right) \sigma_{ig}.$$

- Call this "CCAPM+", because it nests the CCAPM and adds aversion to fluctuations in long-run consumption growth.
- We get CCAPM when  $\gamma = 1/\psi$  (power utility).
- Formula originally derived by Restoy and Weil (1998).

# Long-Run Risk Model

- Bansal and Yaron (2004) "long-run risk" model applies CCAPM+ approach to the equity premium and equity volatility puzzles.
- Initial emphasis on persistent shocks to consumption growth.
- Also adds changing variance, which turns out to be key.
- Bansal, Kiku, and Yaron (2007) boost the effect of changing variance and achieve greater empirical success.
- Beeler and Campbell (2009) take the other side in a debate over the empirical merits of this framework.

# Persistent Consumption Growth

$$r_{w,t+1} - E_t r_{w,t+1} = (\Delta c_{t+1} - E_t \Delta c_{t+1}) + \left(1 - \frac{1}{\psi}\right) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}.$$

$$E_t[r_{i,t+1}] - r_{f,t+1} + \frac{\sigma_i^2}{2} = \gamma \sigma_{ic} + \left(\gamma - \frac{1}{\psi}\right) \sigma_{ig}.$$

Assume shocks to  $c$  and  $g$  are uncorrelated. Then

$$E_t[r_{w,t+1}] - r_{f,t+1} + \frac{\sigma_w^2}{2} = \gamma \sigma_c^2 + \left(\gamma - \frac{1}{\psi}\right) \left(1 - \frac{1}{\psi}\right) \sigma_g^2.$$

The second term is positive if  $\psi > 1$ .

# Persistent Consumption Growth: Another Story

- Other authors have argued that consumption responds sluggishly to shocks because of adjustment costs.
- Thus short-run consumption covariance understates risk.
- Example: Gabaix-Laibson (NBER Macro Annual 2001).
  - ▶ Agents update consumption every  $D$  periods, and the distribution of update times is uniform.
  - ▶ So every period,  $1/D$  of agents adjust.
  - ▶ Household that adjusts at time  $i \in [0, 1]$  can react to fraction  $i$  of information in the period, and affects fraction  $(1 - i)$  of consumption.
  - ▶ Downward bias in sensitivity of consumption to news is

$$\int_0^1 i(1 - i) = \left[ \frac{i^2}{2} - \frac{i^3}{3} \right]_0^1 = \frac{1}{6}.$$

- ▶ Since only  $1/D$  of agents adjust at all, we get  $1/6D$  bias in consumption sensitivity, and  $6D$  bias in estimated risk aversion.

# Persistent Consumption Growth: Another Story

- This story implies that
  - ▶ Aggregate consumption growth is positively autocorrelated as agents gradually adjust to news
  - ▶ Covariance of consumption growth and stock returns is increasing with the horizon
  - ▶ Long-run consumption reveals high true risk, which is obscured at short horizons.
- Empirically, there is some short-run autocorrelation of consumption growth
  - ▶ Probably related to time-averaging of consumption
  - ▶ Working (1960): time-average of a Brownian motion (random walk) is an MA(1) in changes with coefficient 0.25.

# Persistent Consumption Growth: Another Story

- Empirically, stock returns lead consumption growth by one quarter which may result from time-averaging and short delays in consumption
  - ▶ "Beginning of period" timing convention for consumption vs. "end of period" convention
- There is a difference between  $Cov(r_{t+1}, c_{t+h} - c_t)$  and  $Cov(r_{t+1} + \dots + r_{t+h}, c_{t+h} - c_t)$ .
  - ▶ The former increases with  $h$  more strongly than the latter.
  - ▶ The reason is that consumption growth predicts future stock returns negatively.

## Changing Variance

Consider a simple case where  $c_t$  follows a random walk with drift:

$$\Delta c_t = g + \varepsilon_t.$$

The expected return on the wealth portfolio is

$$E_t r_{w,t+1} = -\ln \delta + \frac{g}{\psi} - \frac{\sigma^2}{2} \left(1 - \frac{1}{\psi}\right) (1 - \gamma).$$

Now use the expression

$$p_{it} - d_{it} = \frac{k}{1 - \rho} + E_t \sum_{j=0}^{\infty} \rho^j [\Delta d_{i,t+1+j} - r_{i,t+1+j}].$$

Set  $i = w$ ,  $d_{wt} = c_t$ , and use the above expression for the return on wealth. We get

$$p_{wt} - d_{wt} = \text{constant} + \left(\frac{1}{1 - \rho}\right) \left(1 - \frac{1}{\psi}\right) \left(g + \frac{\sigma^2}{2}(1 - \gamma)\right).$$

## Changing Variance

$$p_{wt} - d_{wt} = \text{constant} + \left( \frac{1}{1-\rho} \right) \left( 1 - \frac{1}{\psi} \right) \left( g + \frac{\sigma^2}{2}(1-\gamma) \right).$$

- Let's hold  $g$  constant while  $\sigma^2$  increases. What does it take for consumption claim price to fall?
- We need  $(1 - 1/\psi)$  and  $(1 - \gamma)$  to have opposite signs, so we need  $\psi$  and  $\gamma$  on the same side of one. Inconsistent with power utility.
- Intuition:
  - ▶ An increase in volatility with unchanged geometric mean consumption growth is an improvement in investment opportunities if  $\gamma < 1$  and a deterioration if  $\gamma > 1$ .
  - ▶ If  $\psi > 1$ , an improvement in investment opportunities causes agents to desire lower consumption relative to wealth, driving up wealth for given consumption. If  $\psi < 1$ , the opposite occurs.
  - ▶ Putting these together, we need  $\psi$  and  $\gamma$  to be on the same side of one to get wealth to fall when volatility increases.

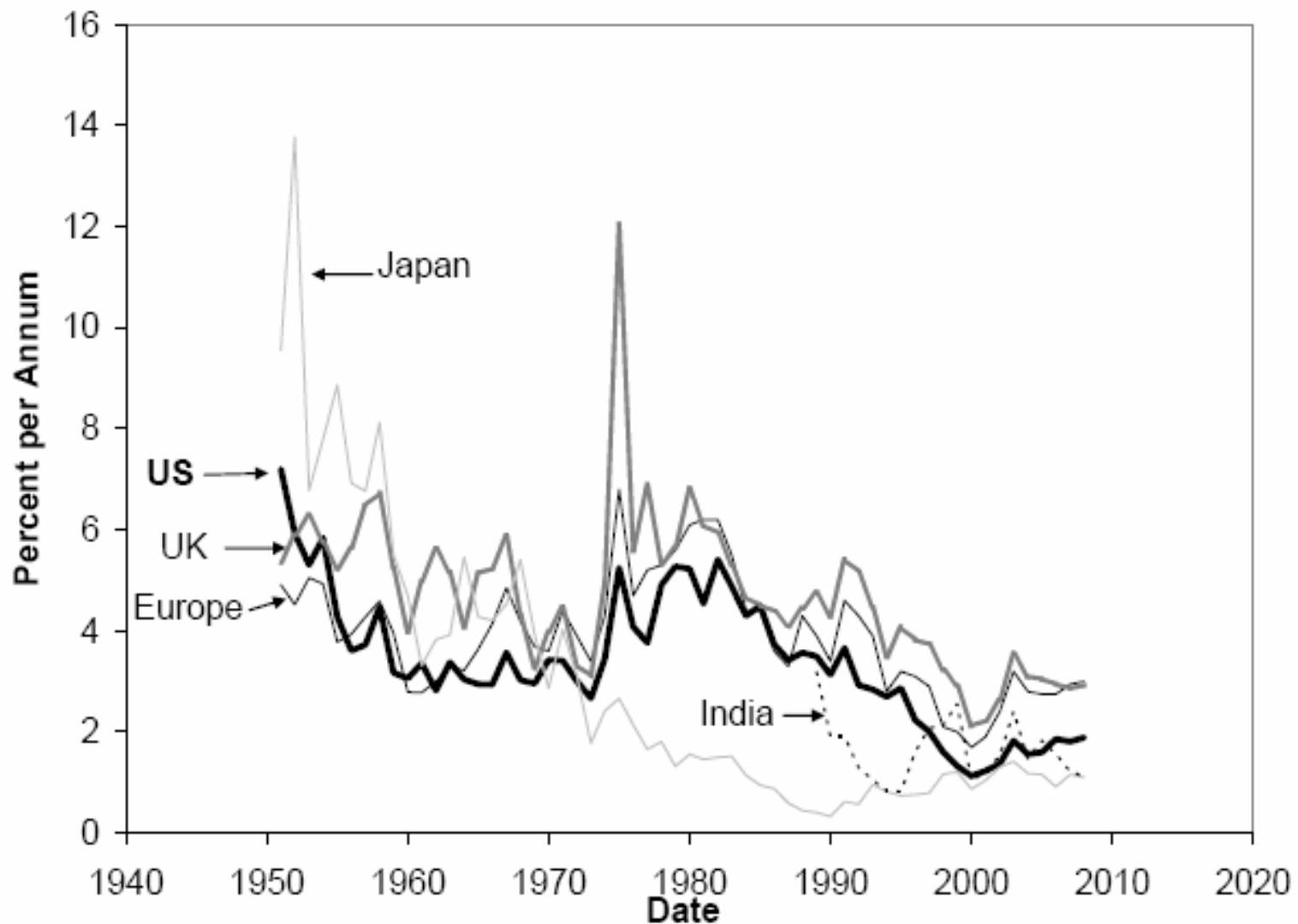
## Changing Variance

$$p_{wt} - d_{wt} = \text{constant} + \left( \frac{1}{1 - \rho} \right) \left( 1 - \frac{1}{\psi} \right) \left( g + \frac{\sigma^2}{2}(1 - \gamma) \right).$$

- Let's hold arithmetic mean consumption growth,  $g + \sigma^2/2$ , constant while  $\sigma^2$  increases. What does it take for consumption claim price to fall?
- We need  $(1 - 1/\psi) > 0$ , that is we need  $\psi > 1$ .
- Intuition:
  - ▶ An increase in volatility with unchanged arithmetic mean consumption growth is a deterioration in investment opportunities for any risk-averse consumer.
  - ▶ If  $\psi > 1$ , a deterioration in investment opportunities causes agents to desire higher consumption relative to wealth, driving down wealth for given consumption.
- The intuition that volatility drives down wealth is the most powerful argument for  $\psi > 1$ .

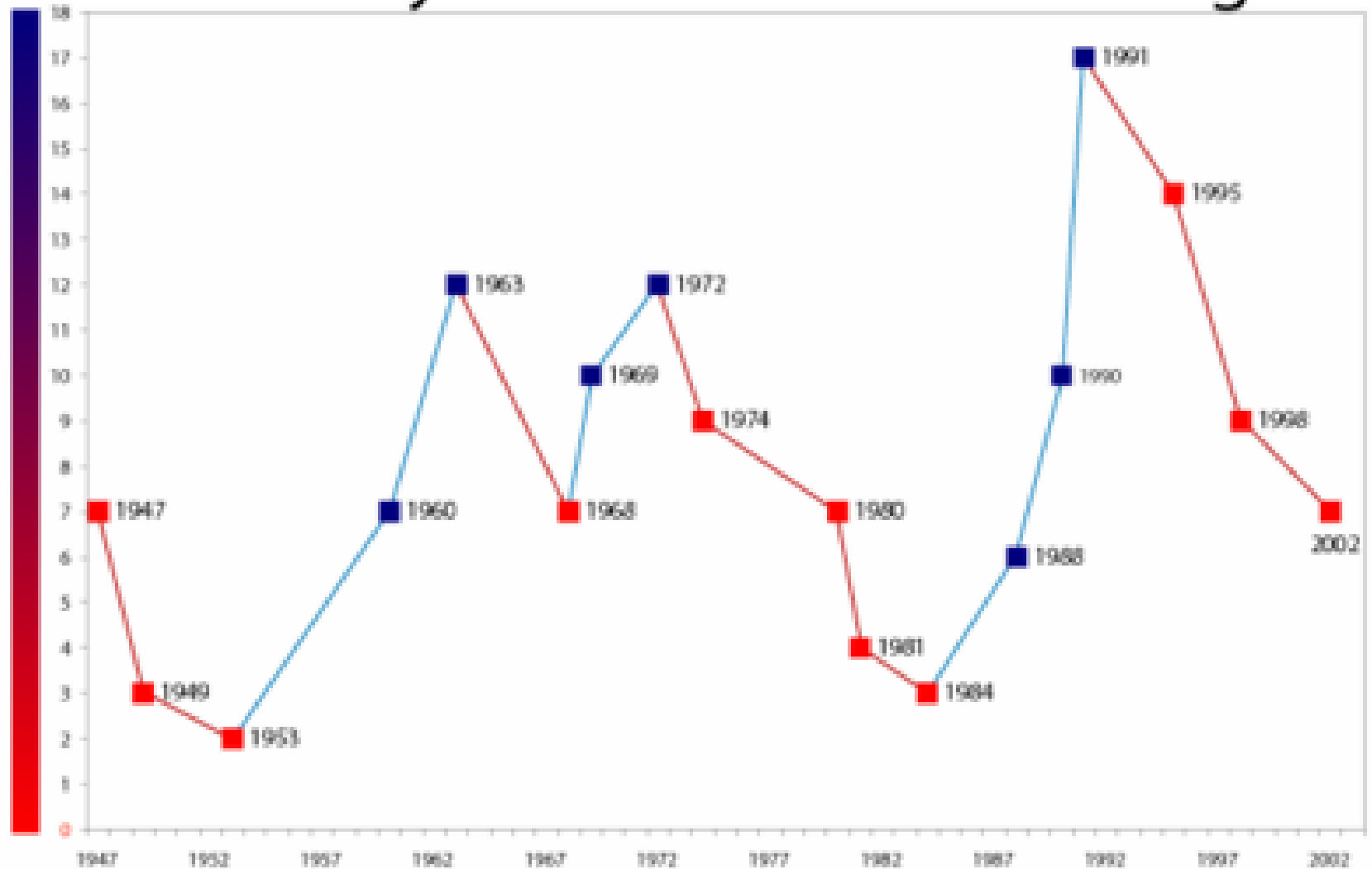
# Asset Volatility and Disaster Risk

- Disaster-risk explanation for equity volatility is that the perceived probability of disaster, or the consequences of disaster for asset holders (the recovery rate or asset "resilience"), change over time.
- If disasters are interpreted as wars, the timing of asset price movements seems off, at least in the last 50 years.
- Changes in resilience are hard to measure.
- An alternative approach: combine disaster risk with limited participation, and interpret disaster as political expropriation.



Source: Robert Shiller, “Low Interest Rates and High Asset Prices”, 2007, using Global Financial Database

# Doomsday Clock: Minutes to Midnight



# Where Next?

- Example: UK 1974 miners' strike, 3-day week, fall of Conservative government
- Spike in labor share (Bottazzi, Pesenti, and van Wincoop, EER 1996), and uncertainty about future of UK capitalism

