Consumption-Based Asset Pricing (3)

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Outline

- Habit formation: $U(C_t, X_t)$ where X_t is some function of past consumption. Modelling issues:
  - **Functional form.** Should U be power in the ratio C/X (Abel 1990) or the difference $C - X$ (Constantinides 1990, Campbell-Cochrane 1999).
  - **Internal vs. external habit.** Does X depend on an agent's own consumption (Constantinides), or on aggregate consumption (Campbell-Cochrane)?
  - **Speed of adjustment.** How quickly does X adjust to C?

- Uninsurable idiosyncratic labor income risk:
  - Conditions under which it makes no difference to the equity premium (Grossman-Shiller 1982)
  - Conditions under which it can make an arbitrarily large difference (Constantinides-Duffie 1996)

Campbell-Cochrane Model

- Functional form: Difference, to get time-varying risk aversion.
- External habit: For simple Euler equations.
- Slow adjustment: To explain long swings in stock prices.
- Issue: How to keep consumption above habit?
  - ▶ Endogenous consumption: Invest PV(habit) in riskless asset
  - ▶ Exogenous consumption: Habit must adjust.

Campbell-Cochrane Model

$$\Delta c_{t+1} = g + \epsilon_{c,t+1}.$$

$$\epsilon_{c,t+1} \sim N(0, \sigma_c^2).$$

Representative agent maximizes

$$E_t \sum_{j=0}^{\infty} \delta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1 - \gamma}.$$

Surplus Consumption Ratio

$$S_t \equiv \frac{C_t - X_t}{C_t}.$$

- Surplus consumption ratio is the fraction of consumption that exceeds habit and is therefore available to generate utility.
- If habit X_t is held fixed as consumption C_t varies, the local coefficient of relative risk aversion is

$$\frac{-C u_{CC}}{u_C} = \frac{\gamma}{S_t}.$$

Log Surplus Consumption Ratio

A well defined model for $s_t \equiv \log(S_t)$ ensures that $S_t > 0$.

$$s_{t+1} = (1 - \varphi)\bar{s} + \varphi s_t + \lambda\left(s_t\right)\epsilon_{c,t+1} \ .$$

- AR(1) with heteroskedastic innovations
- $\lambda\left(s_t\right)$ is the "sensitivity function"
- Parameterization ensures that s_t is the single state variable of the model.

Implied Model for Log Habit

Loglinear approximation around the steady state is

$$x_{t+1} \approx (1-\varphi)\alpha + \varphi x_t + (1-\varphi)c_t = \alpha + (1-\varphi)\sum_{j=0}^{\infty}\varphi^j c_{t-j},$$

- This loglinear process would not keep consumption above habit.
- It might imply implausible behavior of the riskless interest rate.
- Exact model is more complicated and depends on the choice of $\lambda\left(s_t\right)$.

Marginal Utility and the SDF

$$u'(C_t) = (C_t - X_t)^{-\gamma} = S_t^{-\gamma} C_t^{-\gamma}.$$

$$M_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \right)^{-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

$$m_{t+1} = \ln(\delta) - \gamma \Delta s_{t+1} - \gamma \Delta c_{t+1}.$$

$$\text{S.d.}(m) = \gamma \sigma (1 + \lambda(s_t)).$$

- Decreasing $\lambda(s_t)$ function will make risk premia countercyclical. Why?

Riskless Interest Rate

$$r_{t+1}^f = -\log(\delta) + \gamma g - \gamma(1-\varphi)(s_t - \bar{s}) - \frac{\gamma^2 \sigma_c^2}{2} \left[\lambda(s_t) + 1\right]^2 .$$

This has four terms:

1. Impatience
2. Intertemporal substitution w.r.t. consumption growth
3. Intertemporal substitution w.r.t. temporary fluctuations in habit
4. Precautionary savings.

Constant Riskless Interest Rate

Reverse engineer $\lambda(s_t)$ so that

- Precautionary saving and intertemporal substitution exactly offset each other, delivering a constant riskless interest rate.
- Habit is predetermined at and near the steady state $s_t = \bar{s}$.
  - ▶ This also ensures that the derivative of habit with respect to consumption is non-negative.
  - ▶ But note: large shocks to consumption can perversely move habit in the opposite direction (Ljungqvist-Uhlig).

$$s_{\max} = \bar{s} + \frac{1}{2}(1 - \overline{S}^2)$$

$$\lambda(s_t) = \frac{1}{S}\sqrt{1 - 2(s_t - \bar{s})} - 1, \ \ s_t \leq s_{\max}$$
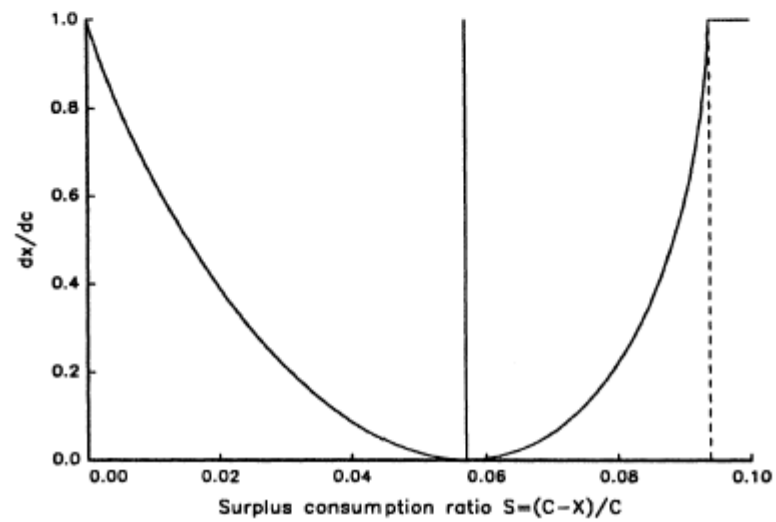
$$\lambda(s_t) = 0, \ \ s_t \geq s_{\max}.$$

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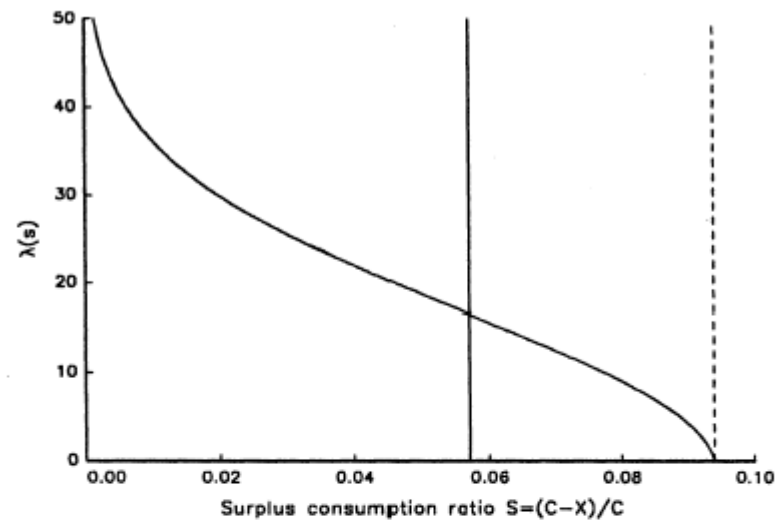
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TABLE 1

Parameter Choices

Parameter	Variable	Value
Assumed:		
Mean consumption growth (%)*	g	1.89
Standard deviation of consumption growth (%)*	σ	1.50
Log risk-free rate (%)*	r^f	.94
Persistence coefficient*	ϕ	.87
Utility curvature	γ	2.00
Standard deviation of dividend growth (%)*	σ_w	11.2
Correlation between Δd and Δc	ρ	.2
Implied:		
Subjective discount factor*	δ	.89
Steady-state surplus consumption ratio	$\bar{S}$	.057
Maximum surplus consumption ratio	S_{max}	.094

* Annualized values, e.g., $12g$, $\sqrt{12}\sigma$, $12r^f$, ϕ^{12}, and δ^{12}, since the model is simulated at a monthly frequency.
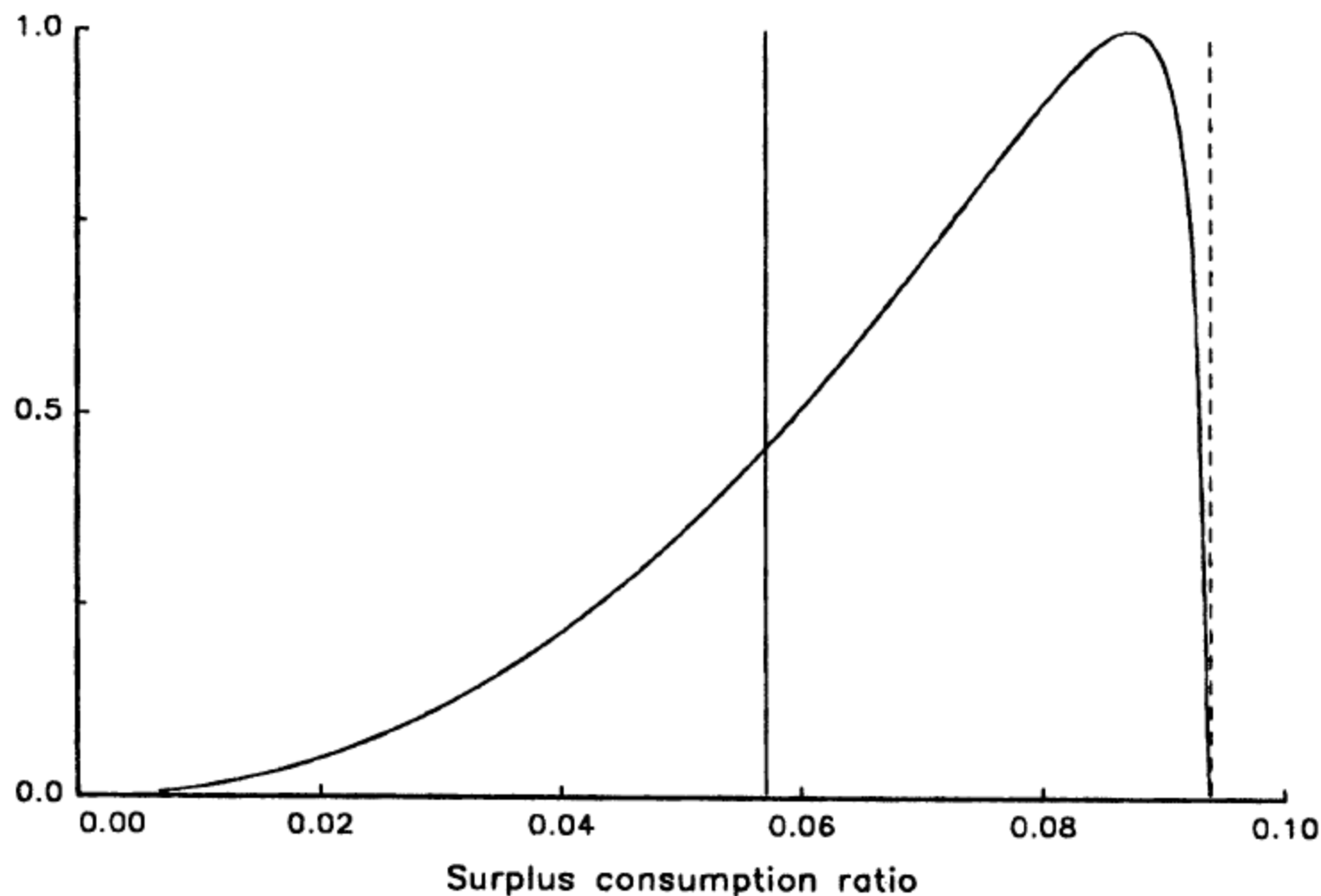
FIG. 2.—Unconditional distribution of the surplus consumption ratio. The solid vertical line indicates the steady-state surplus consumption ratio $\bar{S}$, and the dashed vertical line indicates the upper bound of the surplus consumption ratio S_{max}.

Pricing Consumption Claim

$$\frac{P_t}{C_t}(s_t) = \mathrm{E}_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \left[1 + \frac{P_{t+1}}{C_{t+1}}(s_{t+1}) \right] \right]$$

- Can solve on a grid, or price zero-coupon claims to consumption at a single future date, then add up over time.
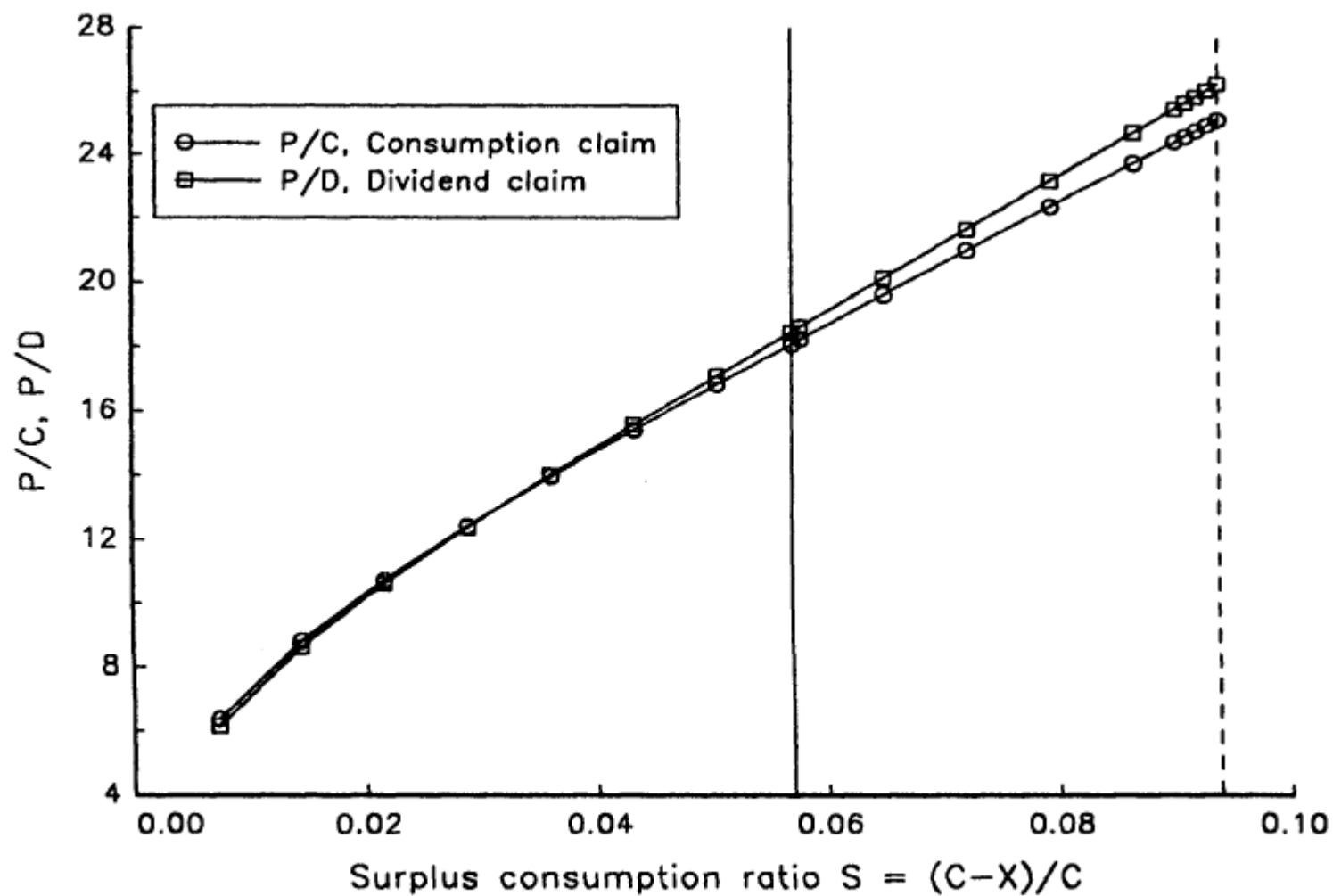- Similar approach works for claims to dividends correlated with consumption.

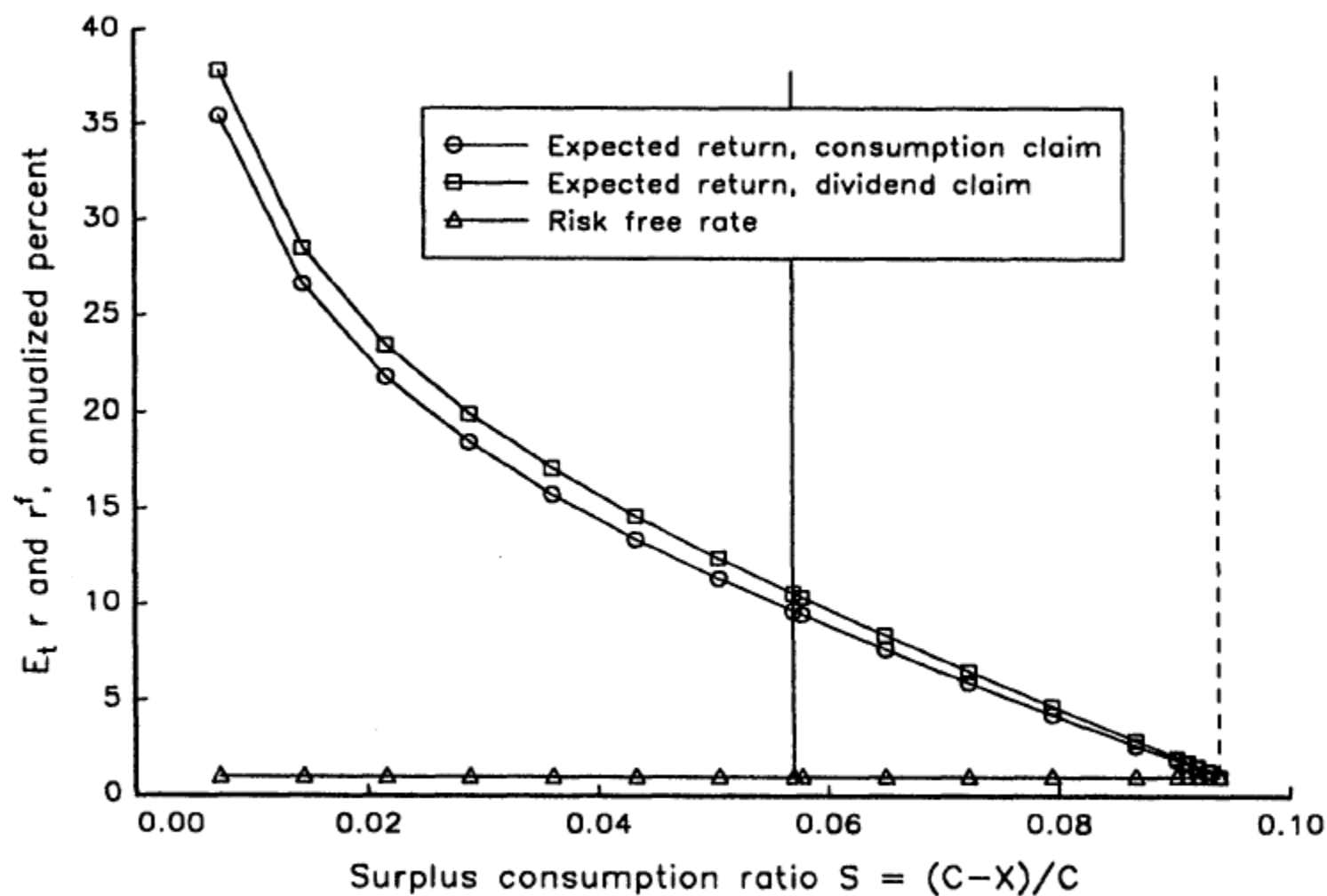FIG. 3.—Price/dividend ratios as functions of the surplus consumption ratio

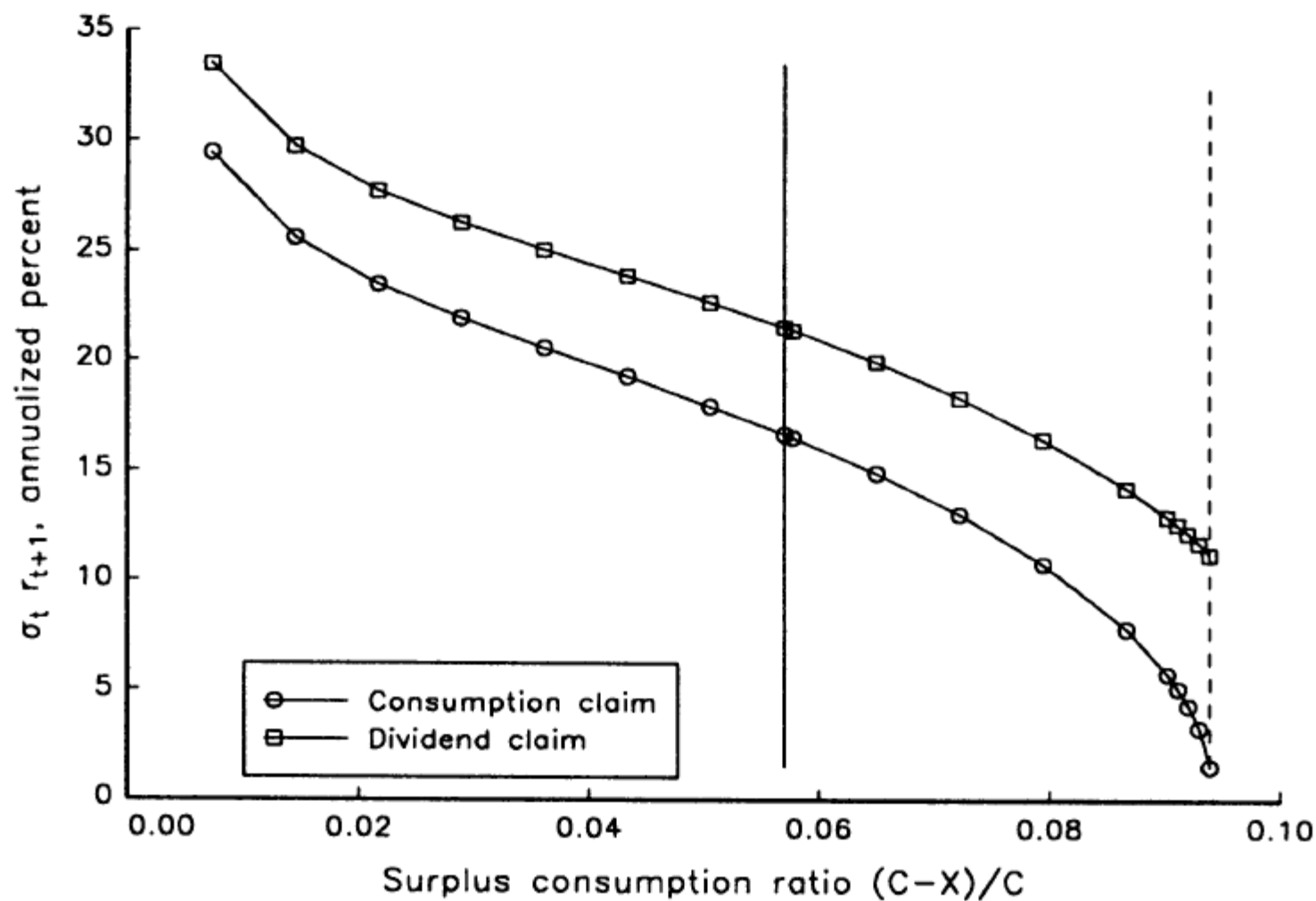FIG. 4.—Expected returns and risk-free rate as functions of the surplus consumption ratio.

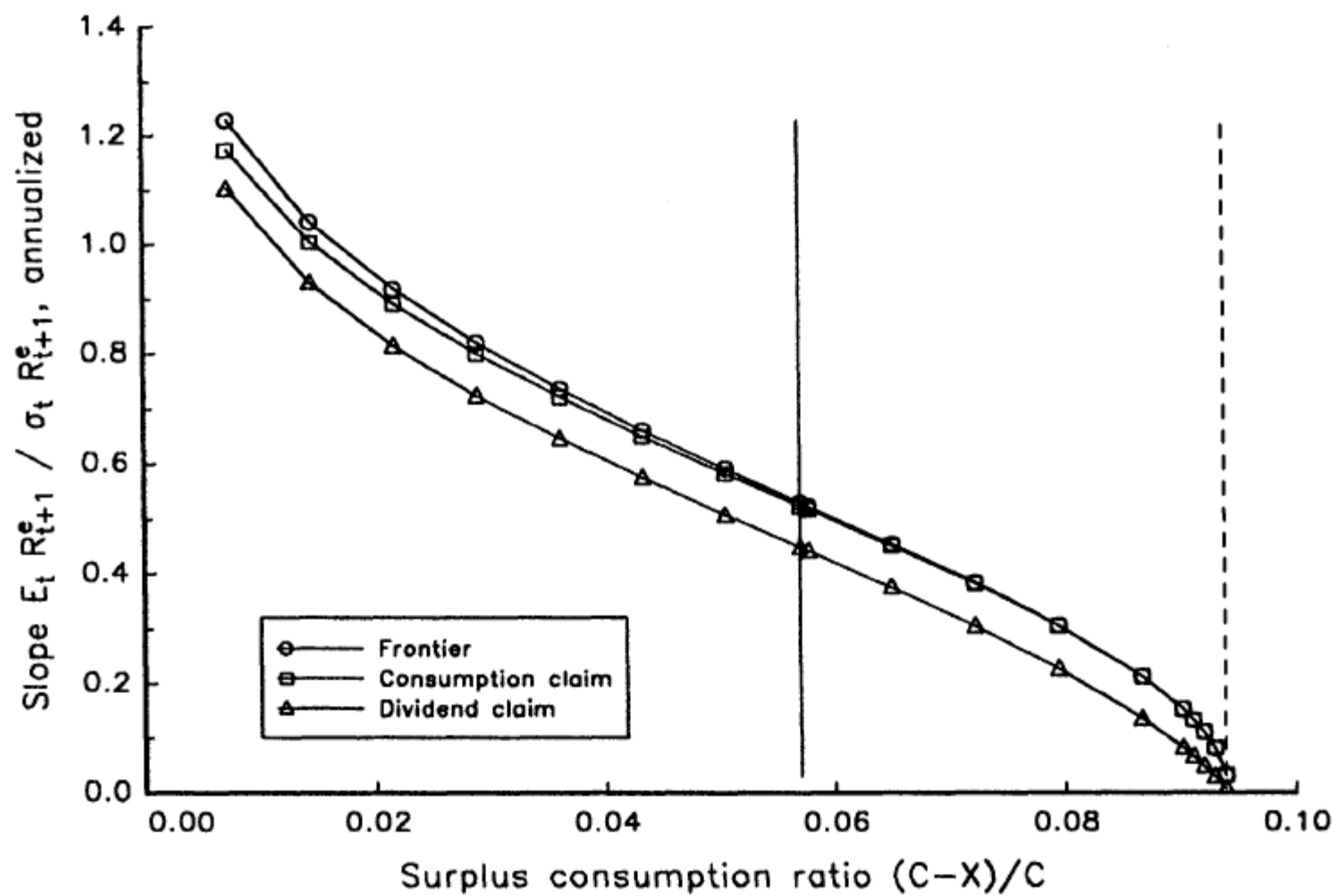FIG. 5.—Conditional standard deviations of returns as functions of the surplus consumption ratio.

FIG. 6.—Sharpe ratios as functions of the surplus consumption ratio

TABLE 2

MEANS AND STANDARD DEVIATIONS OF SIMULATED AND HISTORICAL DATA

Statistic	Consumption Claim	Dividend Claim	Postwar Sample	Long Sample
$E(\Delta c)$	1.89*		1.89	1.72
$\sigma(\Delta c)$	1.22*		1.22	3.32
$E(r^f)$	.094*		.094	2.92
$E(r - r^f)/\sigma(r - r^f)$	.43*	.33	.43	.22
$E(R - R^f)/\sigma(R - R^f)$	.50		.50	
$E(r - r^f)$	6.64	6.52	6.69	3.90
$\sigma(r - r^f)$	15.2	20.0	15.7	18.0
$\exp[E(p - d)]$	18.3	18.7	24.7	21.1
$\sigma(p - d)$	.27	.29	.26	.27

NOTE.—The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency. All returns are annual percentages.
* Statistics that model parameters were chosen to replicate.

TABLE 3

Autocorrelations of Simulated and Historical Data

	Lag (Years)				
Variable and Source	1	2	3	5	7
$p - d$:					
Consumption claim	.87	.76	.66	.51	.39
Dividend claim	.87	.76	.66	.51	.39
Postwar sample	.87	.77	.70	.41	.04
Long sample	.78	.57	.50	.32	.29
$r - r^f$:					
Consumption claim	−.06	−.05	−.04	−.02	−.02
Dividend claim	−.05	−.04	−.03	−.02	−.01
Postwar sample	−.11	−.28	.15	.02	.10
Long sample	.05	−.21	.08	−.14	.11
$\sum_{i=1}^{j} \rho(r_t^e, r_{t-i}^e)$:*					
Consumption claim	−.06	−.11	−.15	−.20	−.26
Dividend claim	−.05	−.09	−.12	−.14	−.18
Postwar sample	−.11	−.39	−.24	.18	.13
Long sample	.05	−.16	−.09	−.28	−.15
$|r|$:					
Consumption claim	.09	.09	.09	.07	.05
Dividend claim	.05	.05	.05	.04	.03
Postwar sample	.08	−.26	−.10	−.08	.05
Long sample	.13	.09	.07	.14	.15

Note.—The model values are based on time-aggregated annual values with a monthly simulation interval. All data are annual.

* Partial sum of return autocorrelations out to lag j.

TABLE 5

Long-Horizon Return Regressions

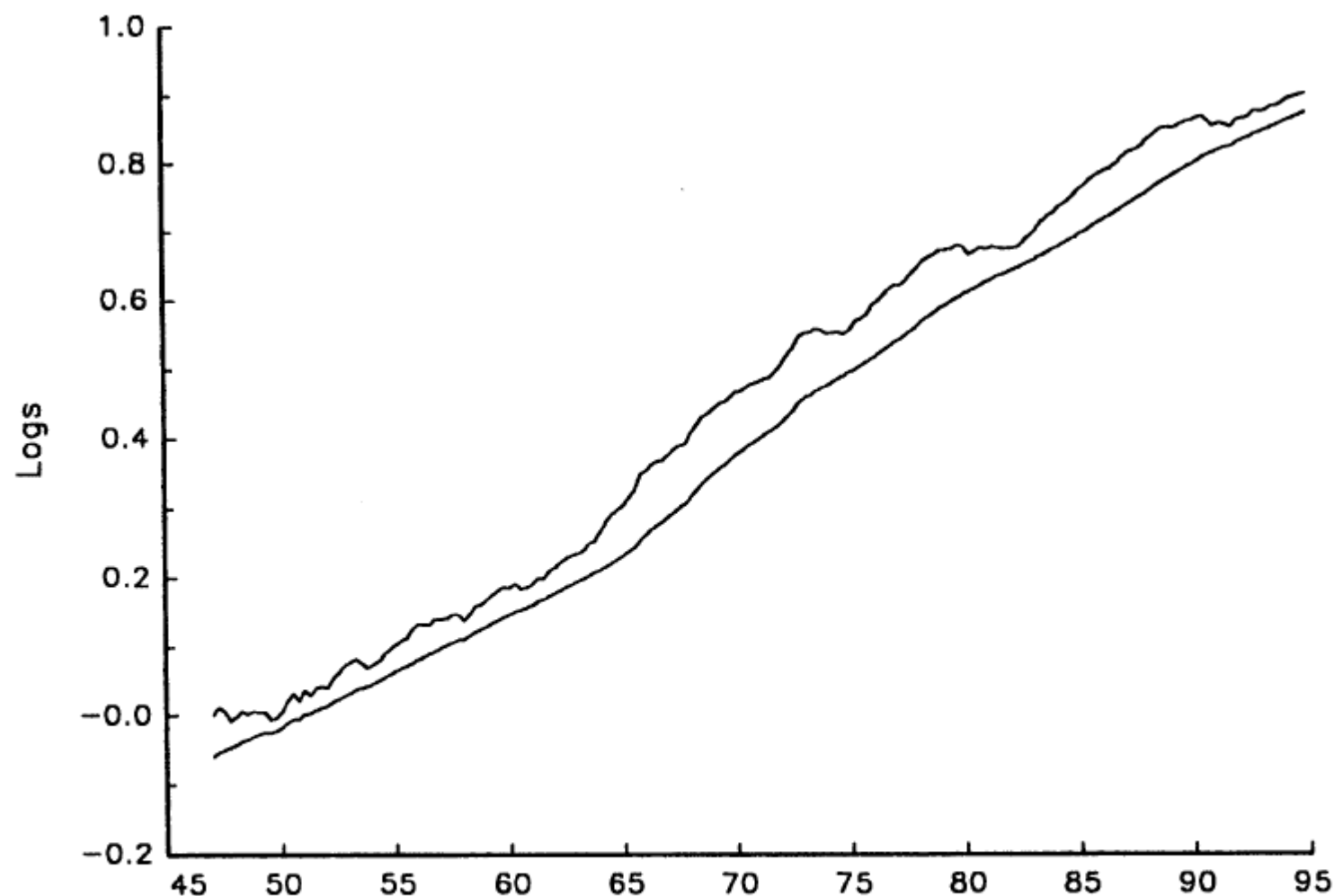| Horizon (Years)	Consumption Claim		Dividend Claim		Postwar Sample		Long Sample	
	$10 \times$ Coefficient	R^2	$10 \times$ Coefficient	R^2	$10 \times$ Coefficient	R^2	$10 \times$ Coefficient	R^2
1	-2.0	.13	-1.9	.08	-2.6	.18	-1.3	.04
2	-3.7	.23	-3.6	.14	-4.3	.27	-2.8	.08
3	-5.1	.32	-5.0	.19	-5.4	.37	-3.5	.09
5	-7.5	.46	-7.3	.26	-9.0	.55	-6.0	.18
7	-9.4	.55	-9.2	.30	-12.1	.65	-7.5	.23

FIG. 8.—Nondurable and services consumption per capita and habit level implied by the model, under the assumption that the surplus consumption ratio starts at the steady state.
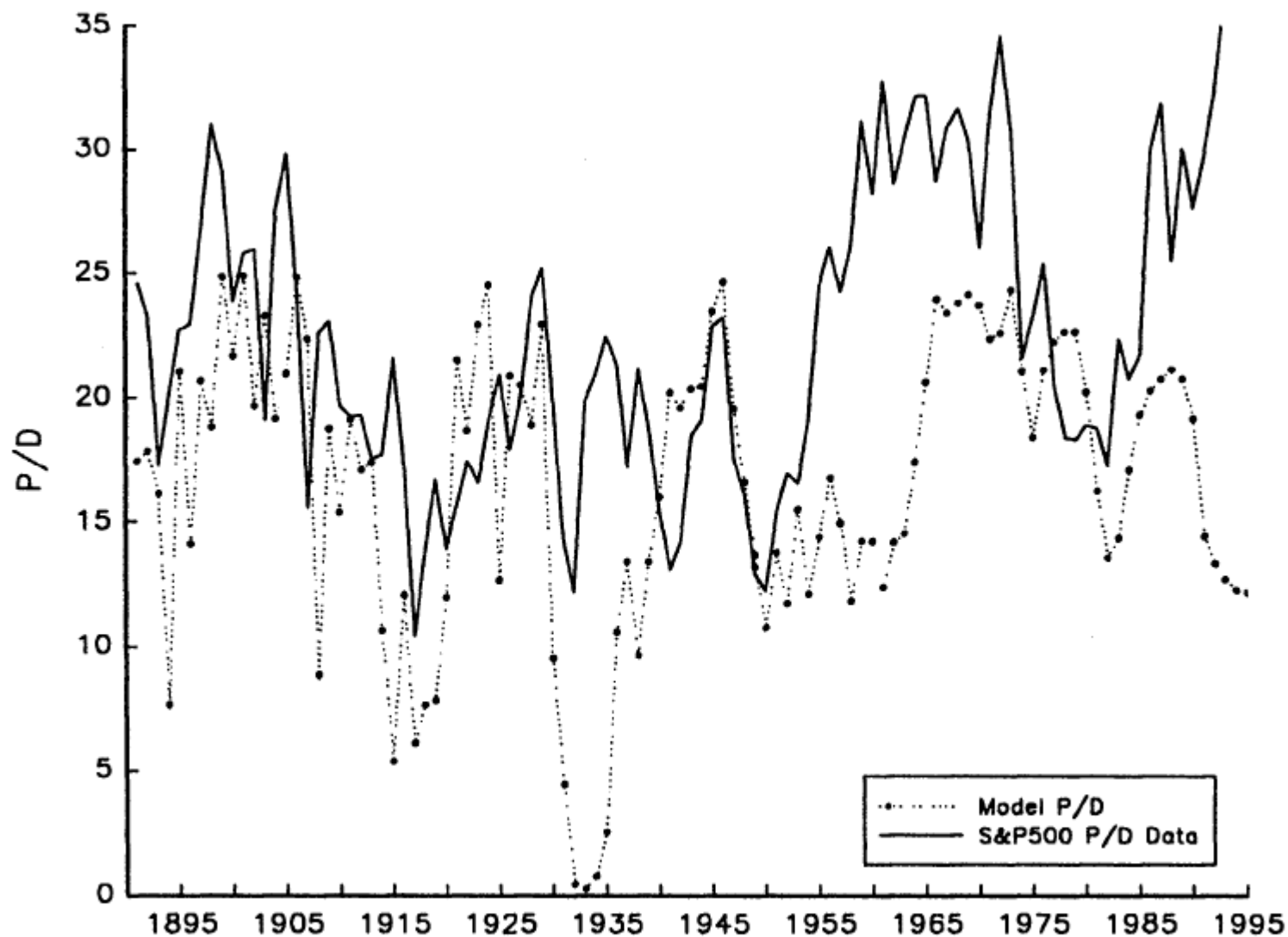
Fig. 9.—Historical price/dividend ratio and model predictions based on the history of consumption.

What Does the Model Achieve?

- The model is intended to address the equity volatility puzzle.
- What does it say about the equity premium puzzle?
- What does it take for any model to solve the equity premium puzzle?

Alternative Interpretations of Habit

Habit formation for the representative investor can arise from

- Identical individual investors with habit formation
- Individual investors with heterogeneous risk aversion coefficients and shifting wealth distribution (Chan-Kogan 2003).
- Participation constraints for some investors (as if these investors have infinite risk aversion).

Heterogeneous Labor Income

- Grossman and Shiller (1982) argued that heterogeneous labor income does not affect risk premia
- Constantinides and Duffie (1996) present a model in which it does.
- What gives?

Grossman-Shiller (1)

Approximate the first-order condition of an individual investor k as

$$
\begin{aligned}
0 &= \mathrm{E}_t \left[(R_{i,t+1} - R_{j,t+1}) \frac{u'(C_{k,t+1})}{u'(C_{kt})} \right] \\
&\approx \mathrm{E}_t \left[(R_{i,t+1} - R_{j,t+1}) \frac{u'(C_{k,t}) + (C_{k,t+1} - C_{kt}) u''(C_{kt})}{u'(C_{kt})} \right] \\
&= \mathrm{E}_t \left[(R_{i,t+1} - R_{j,t+1})(1 - A_k \Delta C_{k,t+1} \right],
\end{aligned}
$$

- A_k is investor k's absolute risk aversion.
- This expression holds exactly when consumption and asset prices are diffusion processes in continuous time.

Grossman-Shiller (2)

Rearranging, we have

$$\frac{1}{A_k} E_t(R_{i,t+1} - R_{j,t+1}) = E_t \left[(R_{i,t+1} - R_{j,t+1}) \Delta C_{k,t+1} \right].$$

Adding up across investors, we have in the aggregate

$$\left(\sum_k \frac{1}{A_k} \right) E_t(R_{i,t+1} - R_{j,t+1}) = E_t \left[(R_{i,t+1} - R_{j,t+1}) \Delta C_{t+1} \right].$$

$$E_t(R_{i,t+1} - R_{j,t+1}) = \left(\sum_k \frac{1}{A_k} \right)^{-1} E_t \left[(R_{i,t+1} - R_{j,t+1}) \Delta C_{t+1} \right].$$

- This is an aggregate consumption-based asset pricing model, using the harmonic mean of individual investors' absolute risk aversion coefficients.
- Idiosyncratic income plays no role.
- How to get away from this conclusion?

Constantinides-Duffie (1)

- Individual investors k have different consumption levels C_{kt}.

- The cross-sectional distribution of individual consumption is lognormal.

- $\Delta c_{k,t+1}$ is cross-sectionally uncorrelated with c_{kt}. Thus there is no steady-state cross-sectional distribution for consumption.

- All investors have power utility with common time discount factor δ and CRRA γ.

- All investors can trade financial assets freely.

Constantinides-Duffie (2)

Each investor's own intertemporal marginal rate of substitution is a valid SDF. Hence the cross-sectional average is too. Write this as

$$M_{t+1}^* \equiv \delta \mathrm{E}_{t+1}^* \left[\left(\frac{C_{k,t+1}}{C_{kt}} \right)^{-\gamma} \right],$$

where E_t^* denotes a cross-sectional expectation: for any X_{kt},

$$\mathrm{E}_t^* X_{kt} = \lim_{K \to \infty} (1/K) \sum_{k=1}^{K} X_{kt}.$$

- $\mathrm{E}_t^* X_{kt}$ can vary over time and need not be lognormally distributed conditional on past information.

Constantinides-Duffie (3)

Cross-sectional lognormality means that the log SDF, $m_{t+1}^* \equiv \log(M_{t+1}^*)$, satisfies

$$m_{t+1}^* = \log(\delta) - \gamma \mathrm{E}_{t+1}^* \Delta c_{k,t+1} + \left(\frac{\gamma^2}{2}\right) \mathrm{Var}_{t+1}^* \Delta c_{k,t+1},$$

where Var_t^* is defined by

$$\mathrm{Var}_t^* X_{kt} = \lim_{K \to \infty} (1/K) \sum_{k=1}^{K} (X_{kt} - \mathrm{E}_t^* X_{kt})^2.$$

Constantinides-Duffie (4)

An economist who knows the underlying preference parameters of investors but does not understand the heterogeneity in this economy might attempt to construct a representative-agent stochastic discount factor, M_{t+1}^{RA}, using aggregate consumption:

$$M_{t+1}^{RA} \equiv \delta \left(\frac{\mathrm{E}_{t+1}^*[C_{k,t+1}]}{\mathrm{E}_t^*[C_{kt}]} \right)^{-\gamma}.$$

The log of this SDF satisfies

$$
\begin{aligned}
m_{t+1}^{RA} &= \log(\delta) - \gamma \mathrm{E}_{t+1}^* \Delta c_{k,t+1} - \left(\frac{\gamma}{2} \right) [\mathrm{Var}_{t+1}^* c_{k,t+1} - \mathrm{Var}_t^* c_{kt}] \\
&= \log(\delta) - \gamma \mathrm{E}_{t+1}^* \Delta c_{k,t+1} - \left(\frac{\gamma}{2} \right) [\mathrm{Var}_{t+1}^* \Delta c_{k,t+1}],
\end{aligned}
$$

- The second equality follows from $c_{k,t+1} = c_{kt} + \Delta c_{k,t+1}$ and the fact that $\Delta c_{k,t+1}$ is cross-sectionally uncorrelated with c_{kt}.

Constantinides-Duffie (5)

The difference between the true and false log SDF is

$$m_{t+1}^* - m_{t+1}^{RA} = \frac{\gamma(\gamma+1)}{2}\text{Var}_{t+1}^*\Delta c_{k,t+1}.$$

- This can have a nonzero mean, helping to explain the riskfree rate puzzle.
- It can have a nonzero time-series variance, helping to explain the equity premium puzzle.
- If the cross-sectional variance of log consumption growth is negatively correlated with the level of aggregate consumption (i.e. if idiosyncratic risk increases in economic downturns), then m_{t+1}^* will be more strongly countercyclical than m_{t+1}^{RA}, driving up the risk premium on cyclical assets.

How Big is the Effect?

The difference between the true and false log SDF is

$$m_{t+1}^* - m_{t+1}^{RA} = \frac{\gamma(\gamma+1)}{2}\text{Var}_{t+1}^*\Delta c_{k,t+1}.$$

- In practice, the cross-sectional volatility of $\Delta c_{k,t+1}$ is fairly stable, so for this effect to be large we still need reasonably high γ.
- How can we reconcile Constantinides-Duffie with Grossman-Shiller?

Interpreting Recent Events

Why did stock prices fall so much more than consumption in the fall of 2008?

- Investors expected slow consumption growth in the future (Bansal-Yaron)
- Investors expected that consumption growth will be volatile for a long time to come (Bansal-Kiku-Yaron)
- Investors became risk-averse because consumption fell close to habit (Campbell-Cochrane)
- Aggressive investors lost wealth relative to cautious investors (Chan-Kogan)
- Idiosyncratic income risk increased (Constantinides-Duffie)